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Asymptotic behavior of Laplacian-energy-like invariant of the 3.6.24 lattice with various boundary conditions

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Abstract

Let G be a connected graph of order n with Laplacian eigenvalues $\mu_1(G) \geq \mu_2(G) \geq \dots \geq \mu_n(G) = 0$. The Laplacian-energy-like invariant of G , is defined as $\mathcal{LEL}(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}$. In this paper, we investigate the asymptotic behavior of the 3.6.24 lattice in terms of Laplacian-energy-like invariant as m, n approach infinity. Additionally, we derive that $M^t(n, m)$, $M^c(n, m)$ and $M^f(n, m)$ have the same asymptotic Laplacian-energy-like invariants.

Keywords: Lattice, Toroidal lattice, Laplacian-energy-like invariant, Laplacian spectrum

Background

Throughout this paper, only undirected and simple connected graphs are considered. Let G be a simple graph with n vertices. The adjacency matrix $A = (a_{ij})$ of G is a $(0, 1)$ -square matrix of order n whose (i, j) -entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. Let $D(G) = \text{diag}(d_1, d_2, \dots, d_n)$ be the diagonal matrix associated to G , where d_i is the degree of vertex v_i . The matrix $L(G) = D(G) - A(G)$ is called Laplacian matrix of G . Let M be a matrix representation of a graph G . For a graph G , let $M = M(G)$ be a corresponding graph matrix defined in a prescribed way. The M -polynomial of G is defined as $\phi_M(G; \lambda) = \det(\lambda I - M)$, where I is the identity matrix. The M -eigenvalues of G are the eigenvalues of M together with their multiplicities. The M -spectrum of G is the multiset of M -eigenvalues of G . In the case of the adjacency matrix (resp. Laplacian matrix), we simply refer to the A -eigenvalues (resp. L -eigenvalues) and A -spectrum (resp. L -spectrum) as the eigenvalues (resp. L -eigenvalues) of G . We denote the eigenvalues of $A(G)$, and $L(G)$ by $\lambda_1(G) \geq \lambda_2(G) \geq \dots \geq \lambda_n(G)$, and $\mu_1(G) \geq \mu_2(G) \geq \dots \geq \mu_n(G) = 0$, respectively. Details on its theory can be found in recent papers (Wang 2014; Liu et al. 2014a, 2016c; Gao et al. 2012; Mohar and Alavi 1991; Liu and Pan 2015a) and the references cited therein.

For the connected graph G , the Laplacian-energy-like invariant of G (Liu and Liu 2008), is defined as $\mathcal{LEL}(G) = \sum_{i=1}^{n-1} \sqrt{\mu_i}$. A general problem of interest in physics, chemistry and mathematics is the calculation of the Laplacian-energy-like invariant of graphs (Wang 2014; Liu et al. 2011), which has now become a popular topic of research.

For more work on $\mathcal{LEL}(G)$, the readers are referred to the most recent papers (Liu and Pan 2015b; Liu et al. 2015, 2016ab, ; Das and Gutman 2014).

Historically in lattice statistics, the hexagonal lattice, 3.12.12 lattice and 3.6.24 lattice have attracted the most attention (Liu and Yan 2013; Ye 2011b; Zhang 2013). Some topological indices of graphs were studied in Li et al. (2015), Yan and Zhang (2009), Ye (2011a), Liu et al. (2014b, 2016d) and Liu and Pan (2016). In fact, Liu et al. have already studied the asymptotic incidence energy (Liu and Pan 2015a) and the Laplacian-energy-like invariant of lattices (Liu et al. 2015).

It is an interesting problem to study the various energies of some lattices with various boundary conditions. W. Wang considered the behavior of Laplacian-energy-like invariant of some graphs in Wang (2014). In present paper, we derive the the Laplacian-energy-like invariant of 3.6.24 lattice via the graph spectrum of the line graph of the subdivision graph of a graph G with the help of computer calculation, which is different from the approach of Wang (2014). Yan et al. investigated the asymptotic behavior of some indices of iterated line graphs of regular graphs in Liu et al. (2016c). Motivated by the above results, in this paper we consider the problem of computations of the $\mathcal{LEL}(G)$ of the 3.6.24 lattice with various boundary conditions.

Preliminaries

We first recall some underlying definitions and lemmas in graph theory.

Some definitions and lemmas

The subdivision graph $s(G)$ of a graph G is obtained from G by deleting every edge uv of G and replacing it by a vertex w of degree 2 that is joined to u and v (see p. 151 of Chartrand and Zhang 2004).

The line graph of a graph G , denoted by $l(G)$, is the graph whose vertices correspond to the edges of G with two vertices of $l(G)$ being adjacent if and only if the corresponding edges in G share a common vertex (Klein and Yi 2012).

Lemma 1 (Gao et al. 2012) *Let G be an r -regular connected graph with n vertices and m edges, then*

$$\begin{aligned}\phi_L(l(G); x) &= (x - 2r)^{m-n} \phi_L(G; x), \\ \phi_L(s(G); x) &= (-1)^m (2 - x)^{m-n} \phi_L(G; x(r + 2 - x)),\end{aligned}$$

where $\phi_L(l(G); x)$ and $\phi_L(s(G); x)$ are the characteristic polynomial for the Laplacian matrix of graphs $l(G)$ and $s(G)$, respectively.

Let a bipartite graph G with a bipartition $V(G) = (U, V)$ is called an (r, s) -semiregular graph if all vertices in U have degree r and all vertices in V have degree s .

Lemma 2 (Mohar and Alavi 1991) *Let G be an (r, s) -semiregular connected graph with n vertices. Then*

$$\phi_L(l(G); x) = (-1)^n (x - (r + s))^{m-n} \phi_L(G; (r + s - x)),$$

where $\phi_L(l(G); x)$ is the Laplacian characteristic polynomial of the line graph $l(G)$ and m is the number of edges of G .

The 3.12.12 and 3.6.24 lattices

The 3.12.12 lattice with toroidal boundary condition (Liu and Yan 2013), denoted $J^t(n, m)$, is illustrated in Fig. 1. Many problems related to the 3.12.12 lattice were considered by physicists (Liu and Yan 2013; Zhang 2013; Liu et al. 2014b). The 3.6.24 lattice with toroidal boundary condition (Zhang 2013), denoted $M^t(n, m)$, is illustrated in Fig. 2.

Based on the constructions of the 3-12-12 and 3.6.24 lattices, we notice that a very important and interesting relationship between 3-12-12 lattice $J^t(n, m)$ and 3-6-24 $M^t(n, m)$ lattice. The relationship is illustrated as follows.

$$J^t(n, m) \xrightarrow{s(J^t(n, m))} s(J^t(n, m)) \xrightarrow{l(s(J^t(n, m)))} M^t(n, m)$$

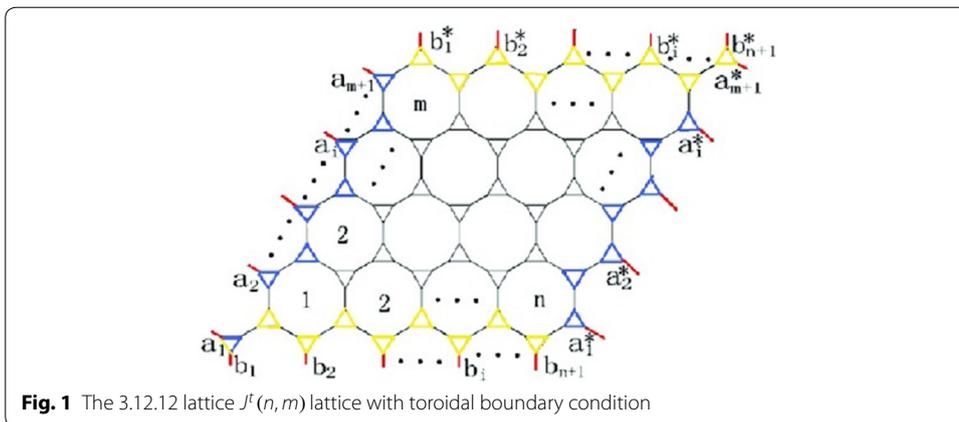


Fig. 1 The 3.12.12 lattice $J^t(n, m)$ lattice with toroidal boundary condition

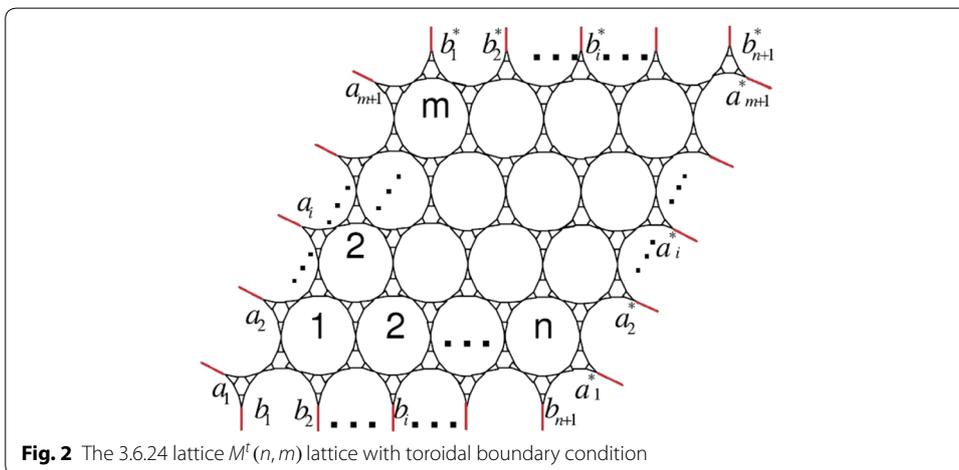


Fig. 2 The 3.6.24 lattice $M^t(n, m)$ lattice with toroidal boundary condition

Main results

In this section, we will explore the Laplacian spectrum of the 3.6.24 lattice with toroidal boundary condition. We begin with the adjacency spectrum of 3.12.12 lattice.

The following adjacency spectrum of 3.12.12 lattice is shown in Liu and Yan (2013).

Theorem 1 (Liu and Yan 2013) *Let $J^t(n, m)$ be the 3.12.12 lattice with toroidal boundary condition. Then the adjacency spectrum is*

$$Spec_A(J^t(n, m)) = \underbrace{\{-2, -2, \dots, -2\}}_{(m+1)(n+1)} \cup \underbrace{\{0, 0, \dots, 0\}}_{(m+1)(n+1)} \cup \left\{ \frac{1 \pm \sqrt{13 \pm 4 \sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}{2} \right\},$$

where $\alpha_i = \frac{2\pi i}{m+1}, \beta_j = \frac{2\pi j}{n+1}, i = 0, 1, \dots, m; j = 0, 1, \dots, n$.

The Laplacian spectrum of the 3.12.12 lattice with toroidal boundary condition is given by the following theorem.

Theorem 2 *Let $J^t(n, m)$ be the 3.12.12 lattice with toroidal boundary condition and $\alpha_i = \frac{2\pi i}{m+1}, \beta_j = \frac{2\pi j}{n+1}, i = 0, 1, \dots, m; j = 0, 1, \dots, n$. Then the Laplacian spectrum is*

$$Spec_L(J^t(n, m)) = \underbrace{\{3, 3, \dots, 3\}}_{(m+1)(n+1)} \cup \underbrace{\{5, 5, \dots, 5\}}_{(m+1)(n+1)} \cup \left\{ \frac{5 \pm \sqrt{13 \pm 4 \sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}{2} \right\}.$$

Proof Consider that $J^t(n, m)$ is a 3-regular graph of order n , then $D(G) = 3I_n$. Hence,

$$L(J^t(n, m)) = 3I_n - A(J^t(n, m)).$$

Define the map $\varphi(\lambda_i) = 3 - \lambda_i$ maps the eigenvalues of $A(J^t(n, m))$ to the eigenvalues of $L(J^t(n, m))$ and can be considered as an isomorphism of the A -spectrum to the corresponding the L -spectrum for $J^t(n, m)$. Based on the fact that G is an r -regular graph with n vertices and $Spec_A(G) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$.

Then $Spec_L(G) = \{r - \lambda_1, r - \lambda_2, \dots, r - \lambda_n\}$. □

Next, we will deduce the Laplacian spectrum of the 3.6.24 lattice $M^t(n, m)$.

Theorem 3 *Let $\mu_1 \geq \mu_2 \dots \geq \mu_{6(m+1)(n+1)} = 0$ are the Laplacian eigenvalues of the 3.12.12 lattice $J^t(n, m)$. Then the Laplacian spectrum of $M^t(n, m)$ is*

$$\begin{aligned}
 \text{Spec}_L(M^t(n, m)) &= \left\{ \underbrace{3, 3, \dots, 3}_{3(m+1)(n+1)}, \underbrace{5, 5, \dots, 5}_{3(m+1)(n+1)} \right\} \\
 &\cup_{i=1}^{6(m+1)(n+1)} \left\{ \frac{5 - \sqrt{25 - 4\mu_i}}{2} \right\} \\
 &\cup_{i=1}^{6(m+1)(n+1)} \left\{ \frac{5 + \sqrt{25 - 4\mu_i}}{2} \right\}.
 \end{aligned}$$

Proof Note that $J^t(n, m)$ has $6(m + 1)(n + 1)$ vertices and $M^t(n, m)$ is the line graph of the subdivision of $J^t(n, m)$ which is a 3-regular graph. That is,

$$M^t(n, m) = l(s(J^t(n, m))). \tag{1}$$

For the convenience of description, we suppose that $s(J^t(n, m))$ has p vertices and q edges. Obviously, $p = 15(m + 1)(n + 1)$ and $q = 18(m + 1)(n + 1)$, respectively. In fact, $s(J^t(n, m))$ is (2, 3)-semi-regular graphs.

By Lemma 2, suppose the graph in equality above is $s(J^t(n, m))$, then the Laplacian characteristic polynomial of $l(s(J^t(n, m)))$ is,

$$\begin{aligned}
 \phi_L(l(s(J^t(n, m))); x) &= (-1)^p(x - 5)^{q-p} \\
 &\times \phi_L(s(J^t(n, m)); (5 - x)),
 \end{aligned} \tag{2}$$

By virtue of Eq. (1), one can immediately obtain that

$$\begin{aligned}
 \phi_L(M^t(n, m); x) &= (-1)^p(x - 5)^{q-p} \\
 &\times \phi_L(s(J^t(n, m)); (5 - x)).
 \end{aligned} \tag{3}$$

On the other hand, note that $J^t(n, m)$ has $9(m + 1)(n + 1)$ edges, it obviously follows from Lemma 1,

$$\begin{aligned}
 \phi_L(s(J^t(n, m)); x) &= (-1)^{9(m+1)(n+1)}(2 - x)^{3(m+1)(n+1)} \\
 &\times \phi_L(J^t(n, m); (x(5 - x))).
 \end{aligned} \tag{4}$$

Consider the term $\phi_L(s(J^t(n, m)); (5 - x))$ in Eq. (3), we replace x with $5 - x$ in Eq. (4), we have

$$\begin{aligned}
 \phi_L(s(J^t(n, m)); (5 - x)) &= (-1)^{9(m+1)(n+1)}(x - 3)^{3(m+1)(n+1)} \\
 &\times \phi_L(J^t(n, m); (x(5 - x))).
 \end{aligned} \tag{5}$$

Combing Eq. (3) with Eq. (5), $p = 15(m + 1)(n + 1)$ and $q = 18(m + 1)(n + 1)$, it holds

$$\begin{aligned} \phi_L(M^t(n, m); x) &= (-1)^{p+9(m+1)(n+1)}(x - 5)^{q-p} \\ &\quad \times (x - 3)^{3(m+1)(n+1)}\phi_L(J^t(n, m); (x(5 - x))) \\ &= (-1)^{24(m+1)(n+1)}((x - 3)(x - 5))^{3(m+1)(n+1)} \\ &\quad \times \phi_L(J^t(n, m); (x(5 - x))). \end{aligned} \tag{6}$$

Note that the roots of $x(5 - x) = \mu_i$ are

$$x_{1,i} = \frac{5 - \sqrt{25 - 4\mu_i}}{2}, \quad x_{2,i} = \frac{5 + \sqrt{25 - 4\mu_i}}{2},$$

where $\mu_1 \geq \mu_2 \dots \geq \mu_{6(m+1)(n+1)} = 0$ are the Laplacian eigenvalues of the 3.12.12 lattice $J^t(m, n)$.

It follows from Eq. (6) that the Laplacian spectrum of $M^t(n, m)$ is

$$\begin{aligned} \text{Spec}_L(M^t(n, m)) &= \left\{ \underbrace{3, 3, \dots, 3}_{3(m+1)(n+1)}, \underbrace{5, 5, \dots, 5}_{3(m+1)(n+1)} \right\} \\ &\quad \bigcup_{i=1}^{6(m+1)(n+1)} \left\{ \frac{5 - \sqrt{25 - 4\mu_i}}{2} \right\} \bigcup_{i=1}^{6(m+1)(n+1)} \left\{ \frac{5 + \sqrt{25 - 4\mu_i}}{2} \right\}, \end{aligned}$$

where μ_i are the Laplacian eigenvalues of the 3.12.12 lattice $J^t(n, m)$.

Theorem 4 Let $A =$

$$\sqrt{5 - \sqrt{15 \pm 2\sqrt{13 \pm 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}},$$

$B =$

$$\sqrt{5 + \sqrt{15 \pm 2\sqrt{13 \pm 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}},$$

and $\alpha_i = \frac{2\pi i}{m+1}, \beta_j = \frac{2\pi j}{n+1}, i = 0, 1, \dots, m; j = 0, 1, \dots, n$. Then

1. The Laplacian-energy-like invariant of $M^t(n, m)$ can be expressed as

$$\begin{aligned} \mathcal{LEL}(M^t(n, m)) &= 3(\sqrt{3} + \sqrt{5})(m + 1)(n + 1) \\ &\quad + \frac{(m + 1)(n + 1)}{\sqrt{2}}(\sqrt{5 - \sqrt{5}} + \sqrt{5 - \sqrt{13}}) \\ &\quad + \frac{(m + 1)(n + 1)}{\sqrt{2}}(\sqrt{5 + \sqrt{5}} + \sqrt{5 + \sqrt{13}}) \\ &\quad + \frac{1}{\sqrt{2}}\sum_{i=0}^m\sum_{j=0}^n A + \frac{1}{\sqrt{2}}\sum_{i=0}^m\sum_{j=0}^n B. \end{aligned}$$

$$2. \mathcal{LEL}(M^t(n, m)) \approx 18.1764(m + 1)(n + 1), \text{ as } m, n \rightarrow \infty.$$

Proof Based on Theorems 2, 3 and the definition of the Laplacian-energy-like invariant, we can arrive at the statement 1 of Theorem 4.

Note that the term A can decompose four terms

$$A = A_1 + A_2 + A_3 + A_4,$$

$$A_1 = \sqrt{5 - \sqrt{15 - 2\sqrt{13 - 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$A_2 = \sqrt{5 - \sqrt{15 - 2\sqrt{13 + 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$A_3 = \sqrt{5 - \sqrt{15 + 2\sqrt{13 - 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$A_4 = \sqrt{5 - \sqrt{15 + 2\sqrt{13 + 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

Similarly,

$$B = B_1 + B_2 + B_3 + B_4,$$

$$B_1 = \sqrt{5 + \sqrt{15 - 2\sqrt{13 - 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$B_2 = \sqrt{5 + \sqrt{15 - 2\sqrt{13 + 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$B_3 = \sqrt{5 + \sqrt{15 + 2\sqrt{13 - 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

$$B_4 = \sqrt{5 + \sqrt{15 + 2\sqrt{13 + 4\sqrt{3 + 2\cos\alpha_i + 2\cos\beta_j + 2\cos(\alpha_i + \beta_j)}}}}$$

Then

$$\begin{aligned} \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\mathcal{L}\mathcal{L}\mathcal{L}(M^t(m, n))}{18(m+1)(n+1)} &= \frac{\sqrt{3} + \sqrt{5}}{6} \\ &+ \frac{\sqrt{2}}{36} \left(\sqrt{5 - \sqrt{5}} + \sqrt{5 - \sqrt{13}} + \sqrt{5 + \sqrt{5}} + \sqrt{5 + \sqrt{13}} \right) \\ &+ \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\sqrt{2}}{36(m+1)(n+1)} \sum_{i=0}^m \sum_{j=0}^n (A_1 + A_2 + A_3 + A_4) \\ &+ \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\sqrt{2}}{36(m+1)(n+1)} \sum_{i=0}^m \sum_{j=0}^n (B_1 + B_2 + B_3 + B_4). \end{aligned}$$

We consider that

$$\begin{aligned} &\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\sqrt{2}}{36(m+1)(n+1)} \sum_{i=0}^m \sum_{j=0}^n (A_1 + A_2 + A_3 + A_4) \\ &= \frac{\sqrt{2}}{36} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} A'_1 dx dy \\ &+ \frac{\sqrt{2}}{36} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} A'_2 dx dy \\ &+ \frac{\sqrt{2}}{36} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} A'_3 dx dy \\ &+ \frac{\sqrt{2}}{36} \cdot \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} A'_4 dx dy \\ &\approx 0.0040, \end{aligned}$$

where

$$A'_1 = \sqrt{5 - \sqrt{15 - 2\sqrt{13 - 4\sqrt{3 + 2\cos x + 2\cos y + 2\cos(x + y)}}}}$$

$$A'_2 = \sqrt{5 - \sqrt{15 - 2\sqrt{13 + 4\sqrt{3 + 2\cos x + 2\cos y + 2\cos(x + y)}}}}$$

$$A'_3 = \sqrt{5 - \sqrt{15 + 2\sqrt{13 - 4\sqrt{3 + 2\cos x + 2\cos y + 2\cos(x + y)}}}}$$

$$A'_4 = \sqrt{5 - \sqrt{15 + 2\sqrt{13 + 4\sqrt{3 + 2\cos x + 2\cos y + 2\cos(x + y)}}}}$$

The above numerical integration values are calculated by using the computer software Matlab.

By a completely similar calculation with software Matlab, we can obtain that

$$\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\sqrt{2}}{36(m+1)(n+1)} \sum_{i=0}^m \sum_{j=0}^n (B_1 + B_2 + B_3 + B_4) \approx 0.0118.$$

Consequently, we have

$$\begin{aligned} & \lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{\mathcal{LEL}(M^t(n, m))}{18(m+1)(n+1)} \\ & \approx \frac{\sqrt{3} + \sqrt{5}}{6} + \frac{\sqrt{2}}{36} (\sqrt{5 - \sqrt{5}} + \sqrt{5 - \sqrt{13}} \\ & \quad + \sqrt{5 + \sqrt{5}} + \sqrt{5 + \sqrt{13}}) \\ & \quad + 0.0040 + 0.0118 \\ & = 1.0098. \end{aligned} \tag{7}$$

The Eq. (7) implies $M^t(n, m)$ has the asymptotic Laplacian-energy-like invariant

$$\mathcal{LEL}(M^t(n, m)) \approx 18.1764(m+1)(n+1),$$

as $m, n \rightarrow \infty$. The theorem thus follows. □

The energy of a graph G with n vertices, denoted by $\mathcal{E}(G)$, is defined by

$$\mathcal{E}(G) = \sum_{i=1}^n |\lambda_i(G)|,$$

where the $\lambda_i(G)$ are the eigenvalues of the adjacency matrix of G . The asymptotic energy per vertex of G (Yan and Zhang 2009) is defined by

$$\lim_{n \rightarrow \infty} \frac{\mathcal{E}(G)}{|V(G_n)|}.$$

Motivated by the above results, we consider the problem of computation of the $\mathcal{LEL}(G)$ per vertex of G (Liu et al. 2015).

Theorem 5 (Liu et al. 2015) *Let $\{G_n\}$ be a sequence of finite simple graphs with bounded average degree such that*

$$\lim_{n \rightarrow \infty} |V(G_n)| = \infty, \quad \lim_{n \rightarrow \infty} \frac{\mathcal{LEL}(G_n)}{|V(G_n)|} = h \neq 0.$$

Let $\{H_n\}$ be a sequence of spanning subgraphs of $\{G_n\}$ such that

$$\lim_{n \rightarrow \infty} \frac{|\{v \in V(H_n) : d_{H_n(v)} = d_{G_n(v)}\}|}{|V(G_n)|} = 1,$$

then

$$\lim_{n \rightarrow \infty} \frac{\mathcal{LEL}(H_n)}{|V(G_n)|} = h.$$

That is, G_n and H_n have the same asymptotic Laplacian-energy-like invariant.

Remark 1 Theorem 5 provides a very effective approach to handle the asymptotic the Laplacian-energy-like invariant of a graph with bounded average degree.

Based on Theorem 5, the following result is straightforward.

Theorem 6 Let $M^t(n, m)$ (resp. $M^c(n, m)$, $M^f(n, m)$) be the toroidal (resp. cylindrical, free) boundary condition of the 3.6.24 lattice. Then

$$\begin{aligned} &\mathcal{LEL}(M^t(n, m)) \\ &= \mathcal{LEL}(M^c(n, m)) \\ &= \mathcal{LEL}(M^f(n, m)) \\ &\approx 18.1764(m + 1)(n + 1). \end{aligned}$$

Remark 2 It follows from Theorems 5 and 6 that the growth rate of the $\mathcal{LEL}(G)$ of the 3.6.24 lattice $M^t(n, m)$ (resp. $M^c(n, m)$, $M^f(n, m)$) with toroidal (resp. cylindrical, free) boundary condition is only dependent on the number of vertices of it.

Conclusions

In this paper, we deduced the formulae expressing the Laplacian-energy-like invariant of the 3.6.24 lattice with various boundary conditions. Moreover, we obtained the explicit asymptotic values of the Laplacian-energy-like invariant by utilizing the analysis methods with the help of software Matlab calculation. In addition, we showed that their growth rates are independent of the structure of $M(n, m)$ and only dependent on the number of vertices of $M(n, m)$. These and some other related issues are very good topics on lattices, which deserves further exploration.

Authors' contributions

J-BL designed the study and completed the article. JC provided the study design. TH and FEA provided scientific advice. All authors read and approved the final manuscript.

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Acknowledgements

The authors would like to express their sincere gratitude to the anonymous referees for many friendly and helpful suggestions, which led to great deal of improvement of the original manuscript. This work is supported by the National Natural Science Foundation of China (grant 11601006, 71571002).

Competing interests

The authors declare that they have no competing interests.

Received: 21 August 2015 Accepted: 9 August 2016

Published online: 24 August 2016

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