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The existence of solutions of q -difference-differential equations

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Abstract

By using the Nevanlinna theory of value distribution, we investigate the existence of solutions of some types of non-linear q -difference differential equations. In particular, we generalize the Rellich–Wittich-type theorem and Malmquist-type theorem about differential equations to the case of q -difference differential equations (system).

Keywords: Transcendental, q -Difference differential equation, Solution, Zero order

Mathematics Subject Classification: 39A 50, 30D 35

Background

In this paper, we shall assume that readers are familiar with the basic theorems and the standard notations of the Nevanlinna value distribution theory of meromorphic functions such as $m(r, f)$, $N(r, f)$, $T(r, f)$, . . ., (see Hayman 1964; Yang 1993; Yi and Yang 1995). For a meromorphic function f , $S(r, f)$ denotes any quantity satisfying $S(r, f) = o(T(r, f))$ for all r outside a possible exceptional set of finite logarithmic measure, $\mathbb{S}(f)$ denotes the family of all meromorphic function $a(z)$ such that $T(r, a) = S(r, f) = o(T(r, f))$, where $r \rightarrow \infty$ outside of a possible exceptional set of finite logarithmic measure. In addition, we denote by $S_1(r, f)$ any quantity satisfying $S_1(r, f) = o(T(r, f))$ for all r on a set F of logarithmic density 1, the logarithmic density of a set F is defined by

$$\limsup_{r \rightarrow \infty} \frac{1}{\log r} \int_{[1, r] \cap F} \frac{1}{t} dt.$$

Throughout this paper, the set F of logarithmic density can be not necessarily the same at each occurrence.

Complex differential equations have attracted many mathematicians, and there are many results about the existence or growth of solutions of differential equations (see He 1981; Laine 1993, 1971; Liao 2015; Tu et al. 2013). In recent, with the development of Nevanlinna theory in complex difference equations (see Barnett et al. 2007; Chiang and Feng 2008; Gundersen et al. 2002; Halburd and Korhonen 2006a, b), there has been an increasing interest in studying difference equations, difference product and q -difference in the complex plane \mathbb{C} , a number of papers (including Chen 2010; Gan 2015; Halburd and Korhonen 2007; Heittokangas et al. 2001; Laine and Yang 2007; Qi and Yang 2015;

Zheng and Chen 2010; Zhang and Korhonen 2010) have focused on the existence and growth of solutions of difference equation.

The following two results had been proved by F. Rellich and H. Wittich, respectively.

Theorem 1 (see He 1981, Rellich). *Let the differential equation be the following form*

$$w'(z) = f(w), \quad (1)$$

If $f(w)$ is transcendental meromorphic function of w , then Eq. (1) has no non-constant entire solution.

Wittich (1955) studied the more general differential equation than Eq. (1) and obtained the following result.

Theorem 2 (see Wittich 1955). *Let*

$$\Phi(z, w) = \sum a_{(i)}(z) w^{i_0} (w')^{i_1} \dots (w^{(n)})^{i_n}$$

be differential polynomial, with coefficients $a_{(i)}(z)$ are polynomial of z . If the right-hand side of the differential equation

$$\Phi(z, w) = f(w), \quad (2)$$

$f(w)$ is the transcendental meromorphic function of w , then the Eq. (2) has no non-constant entire solution.

In the 1980s, Yanagihara and Shimomura extended the above type theorem to the case of difference equations (see Yanagihara 1980, 1983; Shimomura 1981), and obtained the following two results

Theorem 3 (see Shimomura 1981). *For any non-constant polynomial $P(w)$, the difference equation*

$$w(z+1) = P(w(z))$$

has a non-trivial entire solution.

Theorem 4 (see Yanagihara 1980). *For any non-constant rational function $R(w)$, the difference equation*

$$w(z+1) = R(w(z))$$

has a non-trivial meromorphic solution in the complex plane.

Conclusions and our main results

In the present paper, we mainly study the above Rellich–Wittich-type theorem of q -difference differential equation (system).

Definition 5 We call the equation a q -difference differential equation (system) if a equation (system) contains the q -difference term $f(qz)$ and differential term $f'(z)$ of one function $f(z)$ at the same time.

We consider the system of q -difference differential equation of the form

$$\begin{cases} \Omega_1(z, w_1) := \sum_{J_1} a_{J_1}(z) \prod_{j=1}^{n_1} (w_1^{(j)}(q_j z))^{i_j} = P_s[f(w_2)], \\ \Omega_2(z, w_2) := \sum_{J_2} b_{J_2}(z) \prod_{j=1}^{n_2} (w_2^{(j)}(q_j z))^{i_j} = P_t[f(w_1)], \end{cases} \tag{3}$$

where $a_{J_1}(z), b_{J_2}(z)$ are polynomials of z and $q \in \mathbb{C} \setminus \{0\}$, $P_m[f]$ is a polynomial of f of degree m ,

$$P_m[f] = d_m(z)f^m + d_{m-1}(z)f^{m-1} + \dots + d_0(z),$$

and $d_m(z), \dots, d_0(z)$ are polynomials of z , and obtain the following results.

Theorem 6 For system (3), if $s \geq 1, t \geq 1$ and f is a transcendental meromorphic function, then the system (3) has no non-constant transcendental entire solutions (w_1, w_2) with zero order.

Remark 7 Under the assumptions of Theorem 6, the system of q -difference differential equation

$$\begin{cases} \sum_{J_1} a_{J_1}(z) \prod_{j=1}^{n_1} (w_1^{(j)}(q_j z))^{i_j} = \frac{P_{s_2}[f(w_2)]}{Q_{t_2}[f(w_2)]}, \\ \sum_{J_2} b_{J_2}(z) \prod_{j=1}^{n_2} (w_2^{(j)}(q_j z))^{i_j} = \frac{P_{s_1}[f(w_1)]}{Q_{t_1}[f(w_1)]}, \end{cases}$$

has no non-constant transcendental entire solutions (w_1, w_2) with zero order, where $s_1, s_2 \geq 1$ and $P_{s_i}[f]$ and $Q_{t_i}[f]$ are irreducible polynomials in f .

If $s = t$ and $w_1 = w_2$, we can get the following theorem easily

Theorem 8 Let

$$\Omega(z, w) := \sum_J a_J(z) \prod_{j=1}^n (w^{(j)}(q_j z))^{i_j} = P_s[f(w)], \tag{4}$$

if $s \geq 1$ and f is a transcendental meromorphic function, then the system (4) has no non-constant transcendental entire solution with zero order.

From Remark 7, we have

Remark 9 Let $s \geq 1$ and f be a transcendental meromorphic function, then the equation

$$\sum_J a_J(z) \prod_{j=1}^n (w^{(j)}(q_j z))^{i_j} = \frac{P_s[f(w)]}{Q_t[f(w)]},$$

has no non-constant transcendental entire solution with zero order, where $P_s[f]$ and $Q_t[f]$ are irreducible polynomials in f .

As we know, it is very interest problem about the Malmquist theorem of differential equations, Laine (1993) gave the following results

Theorem 9 (see Laine 1993). *Let*

$$(w'(z))^n = R(z, w), \tag{5}$$

where $R(z, w)$ is defined as

$$R(z, w) = \frac{\sum_{i=0}^k a_i(z)}{\sum_{j=0}^l b_j(z)}.$$

If Eq. (5) has transcendental meromorphic solution, then there will be $l = 0$ and $k \leq 2n$.

Theorem 10 (see Laine 1993). *Let*

$$\sum a_{(i)}(z)w^{i_0}(w')^{i_1} \dots (w^{(n)})^{i_n} = R(z, w), \tag{6}$$

where $R(z, w)$ is defined as in Theorem 9. If Eq. (6) has transcendental meromorphic solution, then there will be $l = 0$ and $k \leq \min\{\Delta, \lambda + \mu(1 - \Theta(\infty))\}$, where

$$\Delta = \max \left\{ \sum_{\alpha=0}^n (1 + \alpha) i_{\alpha} \right\}, \quad \lambda = \max \left\{ \sum_{\alpha=0}^n i_{\alpha} \right\},$$

and

$$\mu = \max \left\{ \sum_{\alpha=0}^n \alpha i_{\alpha} \right\}, \quad \Theta(\infty) = 1 - \limsup_{r \rightarrow \infty} \frac{\bar{N}(r, w)}{T(r, w)}.$$

Recently, there were a number of papers focused on the Malmquist-type theorem of the complex difference equations. Ablowitz et al. (2000) proved some results on the classical Malmquist-type theorem of the complex difference equations by applying Nevanlinna theory. Besides, Gao, Xu and Li also studied some systems of complex difference equation and obtained some more precise results related to Malmquist-type theorem (see Gao 2012a, b, c; Li and Gao 2015; Xu et al. 2013, 2015; Xu and Xuan 2015). In this paper, we mainly study the q -difference differential equation about the Malmquist-type theorem, and obtain the following theorem.

Theorem 11 *Let*

$$(w'(qz))^n = R(z, w), \tag{7}$$

where $R(z, w)$ is defined as

$$R(z, w) = \frac{P(z, w)}{Q(z, w)} = \frac{\sum_{i=0}^k a_i(z)w^i}{\sum_{j=0}^l b_j(z)w^j},$$

$P(z, w)$ and $Q(z, w)$ are irreducible polynomials in w , coefficients $a_i(z), b_j(z)$ are rational functions of z . If Eq. (7) exists transcendental meromorphic solutions with zero order, then we also think that $l = 0$ and $k \leq 2n$.

Similar to the proof of Theorem 11, we can get the following corollary easily.

Corollary 12 *Let*

$$\sum a_{(i)}(z)w^{i_0}(w')^{i_1}(q_1z) \cdots (w^{(n)}(q_nz))^{i_n} = R(z, w), \tag{8}$$

where $R(z, w)$ is defined as in Theorem 11. If Eq. (8) has transcendental meromorphic solution of zero order, then there will be $l = 0$ and $k \leq \min\{\Delta, \lambda + \mu(1 - \Theta(\infty))\}$, where Δ, λ and μ are stated as in Theorem 10.

Some Lemmas

Lemma 13 (Valiron-Mohon'ko, Laine 1993). Let $f(z)$ be a meromorphic function. Then for all irreducible rational functions in f ,

$$R(z, f(z)) = \frac{\sum_{i=0}^m a_i(z)f(z)^i}{\sum_{j=0}^n b_j(z)f(z)^j},$$

with meromorphic coefficients $a_i(z), b_j(z)$, the characteristic function of $R(z, f(z))$ satisfies

$$T(r, R(z, f(z))) = dT(r, f) + O(\Psi(r)),$$

where $d = \max\{m, n\}$ and $\Psi(r) = \max_{i,j}\{T(r, a_i), T(r, b_j)\}$.

Lemma 14 (Zhang and Korhonen 2010, Theorem 1 and Theorem 3) Let $f(z)$ be a transcendental meromorphic function of zero order and q be a nonzero complex constant. Then

$$T(r, f(qz)) = (1 + o(1))T(r, f(z))$$

and

$$N(r, f(qz)) = (1 + o(1))N(r, f(z)),$$

on a set of logarithmic density 1.

Lemma 15 (see Barnett et al. 2007). Let $f(z)$ be a nonconstant zero-order meromorphic function and $q \in \mathbb{C} \setminus \{0\}$. Then

$$m\left(r, \frac{f(qz)}{f(z)}\right) = S(r, f),$$

on a set of logarithmic density 1 for all r outside a possible exceptional set of logarithmic density 0.

Lemma 16 (see Yi and Yang 1995, p. 37 or Yang 1993). Let $f(z)$ be a nonconstant meromorphic function in the complex plane and l be a positive integer. Then

$$N(r, f^{(l)}) = N(r, f) + l\bar{N}(r, f), \quad T(r, f^{(l)}) \leq T(r, f) + l\bar{N}(r, f) + S(r, f).$$

Lemma 17 *Let $q \in \mathbb{C} \setminus \{0\}$ and $f(z)$ be a nonconstant meromorphic function with zero order. Then for any positive finite integer k , we have*

$$m\left(r, \frac{f^{(k)}(qz)}{f(z)}\right) = S_1(r, f),$$

and

$$m\left(r, f^{(k)}(qz)\right) \leq m(r, f) + S_1(r, f).$$

Proof It follows from Lemma 15 that

$$m\left(r, \frac{f^{(k)}(qz)}{f(z)}\right) \leq m\left(r, \frac{f^{(k)}(qz)}{f(qz)}\right) + m\left(r, \frac{f(qz)}{f(z)}\right) = S_1(r, f).$$

Moreover, we have

$$m\left(r, f^{(k)}(qz)\right) = m\left(r, \frac{f^{(k)}(qz)}{f(z)}f(z)\right) \leq m(r, f) + S_1(r, f).$$

This completes the proof of Lemma 17. □

The Proof of Theorem 6

Suppose that $w_i (i = 1, 2)$ be non-constant entire functions solutions of system (3) with zero order. Suppose $i = 1$, let $E_1 = \{z : |w_1(z)| > 1\}$ and $E_2 = \{z : |w_1(z)| \leq 1\}$, then we have

$$\begin{aligned} |\Omega_1(z, w_1)| &= \left| \sum_{J_1} a_{J_1}(z)(w_1(z))^{\lambda_i} \left(\frac{w'_1(q_1z)}{w_1(z)}\right)^{i_1} \cdots \left(\frac{w'_1(q_{n_1}z)}{w_1(z)}\right)^{i_{n_1}} \right| \\ &\leq \begin{cases} |w_1(z)|^\lambda \sum_{J_1} |a_{J_1}(z)| \left|\frac{w'_1(q_1z)}{w_1(z)}\right|^{i_1} \cdots \left|\frac{w'_1(q_{n_1}z)}{w_1(z)}\right|^{i_{n_1}}, & \text{if } z \in E_1, \\ \sum_{J_1} |a_{J_1}(z)| \left|\frac{w'_1(q_1z)}{w_1(z)}\right|^{i_1} \cdots \left|\frac{w'_1(q_{n_1}z)}{w_1(z)}\right|^{i_{n_1}}, & \text{if } z \in E_2, \end{cases} \end{aligned}$$

where $\lambda = \max\{\lambda_i\}$, $\lambda_i = i_1 + \cdots + i_{n_1}$. It follows from Lemma 15 and 17 that

$$m(r, \Omega_1(z, w_1)) = \frac{1}{2\pi} \left(\int_{E_1} + \int_{E_2} \right) \log^+ |\Omega_1(z, w_1)| d\theta \leq \lambda m(r, w_1) + S_1(r, w_1).$$

And since $w_1(z)$ is a non-constant entire function, we have $N(r, w_1) = 0$. Thus, we have $N(r, \Omega_1(z, w_1)) = 0$ and

$$T(r, \Omega_1) = m(r, \Omega_1) \leq \lambda m(r, w_1) + S_1(r, w_1). \tag{9}$$

Similarly, we have

$$T(r, \Omega_2) = m(r, \Omega_2) \leq \eta m(r, w_2) + S_1(r, w_2), \tag{10}$$

where $\eta = \max\{\eta_i\}$, $\eta_i = i_1 + \cdots + i_{n_2}$.

Since $P_s[f(w_2)]$ is a polynomial of $f(w_2)$, we can take a complex constant α such that

$$P_s[f(w_2)] - \alpha = [f(w_2) - \beta_1] \cdots [f(w_2) - \beta_s],$$

where β_1, \dots, β_s are complex constants, and there at least exists a constant $\beta \in \{\beta_1, \dots, \beta_s\}$ which is not Picard exceptional value of $f(w_2)$. Let $\{\xi_j, j = 1, 2, \dots, p_2\}$ be the zeros of $f(w_2) - \beta$, where p_2 is any positive integer with $p_2 \geq 1$. Then it follows

$$\sum_{j=1}^{p_2} N\left(r, \frac{1}{w_2 - \xi_j}\right) \leq N\left(r, \frac{1}{f(w_2) - \beta}\right) \leq N\left(r, \frac{1}{P_s[f(w_2)] - \alpha}\right). \tag{11}$$

Thus, by using the second main theorem and (10), (11), we can get that

$$\begin{aligned} (p_2 - 2)T(r, w_2) &\leq \sum_{j=1}^{p_2} N\left(r, \frac{1}{w_2 - \xi_j}\right) + S(r, w_2) \\ &\leq N\left(r, \frac{1}{P_s[f(w_2)] - \alpha}\right) + S(r, w_2) \\ &\leq T(r, P_s[f(w_2)]) + S(r, w_2) \\ &\leq T(r, \Omega_1(z, w_1)) + S(r, w_2) \\ &\leq \lambda T(r, w_1) + S_1(r, w_1) + S_1(r, w_2). \end{aligned} \tag{12}$$

Similarly, there exists any positive integer $p_1 (\geq 1)$ such that

$$(p_1 - 2)T(r, w_1) \leq \eta T(r, w_2) + S_1(r, w_1) + S_1(r, w_2). \tag{13}$$

It follows from (12) and (13) that

$$[(p_1 - 2)(p_2 - 2) - \lambda\eta]T(r, w_i) \leq S_1(r, w_1) + S_1(r, w_2), \tag{14}$$

Since $w_i (i = 1, 2)$ are transcendental and p_1, p_2 are arbitrary, we can get a contradiction with (4). Hence, we complete the proof of Theorem 6.

The Proof of Theorem 11

We firstly choose a constant $a \in \mathbb{C}$ such that $P(z, a) \neq 0$ and $Q(z, a) \neq 0$, then (7) can be rewritten as

$$(w'(qz))^n = \frac{P(z, a) + A_1(z)(w - a) + \cdots + A_k(z)(w - a)^k}{Q(z, a) + B_1(z)(w - a) + \cdots + B_l(z)(w - a)^l}, \tag{15}$$

where $A_1(z), \dots, A_k(z), B_1(z), \dots, B_l(z)$ are all rational functions. Let $\varphi(z) = \frac{1}{w(z) - a}$ that is, $w(z) = \frac{1}{\varphi(z)} + a$ and

$$(w'(qz))^n = (-1)^n \varphi(qz)^{-2n} (\varphi'(qz))^n. \tag{16}$$

Hence, it follows from (15) and (16) that

$$\begin{aligned}
 (\varphi'(qz))^n &= (-1)^{-n} \varphi(qz)^{2n} (w'(qz))^n \\
 &= (-1)^{-n} \varphi(qz)^{2n} \frac{P(z, a) + A_1(z)\varphi(z)^{-1} + \dots + A_k(z)\varphi(z)^{-k}}{Q(z, a) + B_1(z)\varphi(z)^{-1} + \dots + B_l(z)\varphi(z)^{-l}} \\
 &=: \varphi(qz)^{2n} \varphi(z)^{l-k} \frac{\sum_{i=0}^k \tilde{a}_i(z)\varphi(z)^i}{\sum_{i=0}^l \tilde{b}_i(z)\varphi(z)^i} \\
 &= \frac{\tilde{P}(z, \varphi(z))}{\tilde{Q}(z, \varphi(z))} = \tilde{R}(z, \varphi(z)), \tag{17}
 \end{aligned}$$

where $\tilde{a}_i(z) = (-1)^{-n} P(z, a) \neq 0, \tilde{b}_i(z) = Q(z, a) \neq 0$.

Suppose that $w(z)$ is a transcendental meromorphic solution of equation (7) with zero order, then $\varphi(z) = \frac{1}{w(z)-a}$ is also a transcendental meromorphic solution of Eq. (17). We will discuss two cases as follows.

If $2n + l - k \leq 0$, then $\deg_{\varphi} \tilde{P}(z, \varphi) = k$ and $\deg_{\varphi} \tilde{Q}(z, \varphi) = l - (2n + l - k) = k - 2n$. It follows by Lemma 13 that

$$T(r, \tilde{R}(z, \varphi(z))) = kT(r, \varphi) + S(r, \varphi).$$

And by Lemmas 13–17, we have

$$\begin{aligned}
 T(r, (\varphi'(qz))^n) &= nT(r, \varphi'(qz)) \\
 &\leq nN(r, \varphi(qz)) + n\bar{N}(r, \varphi'(qz)) + nm(r, \varphi'(qz)) \\
 &\leq 2nT(r, \varphi(qz)) + S(r, \varphi) \\
 &\leq 2nT(r, \varphi) + S_1(r, \varphi).
 \end{aligned}$$

Thus, it follows

$$kT(r, \varphi) \leq 2nT(r, \varphi) + S_1(r, \varphi), \tag{18}$$

which implies $k \leq 2n$. Since $2n + l - k \leq 0$ and $l \geq 0$, then we have $l = 0$.

If $2n + l - k \geq 0$, then $\deg_{\varphi} \tilde{P}(z, \varphi) = 2n + l$ and $\deg_{\varphi} \tilde{Q}(z, \varphi) = l$. It follows by Lemma 13 that

$$T(r, \tilde{R}(z, \varphi(z))) = (2n + l)T(r, \varphi) + S(r, \varphi).$$

Similar to the argument as in above, we can get $l = 0$ and $k \leq 2n$.

This completes the proof of Theorem 11.

Authors' contributions

XLW, HW and HX completed the main part of this article. All authors read and approved the final manuscript.

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Competing interests

The authors declare that they have no competing interests.

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