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Multi-choice stochastic transportation problem involving general form of distributions

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Abstract

Many authors have presented studies of multi-choice stochastic transportation problem (MCSTP) where availability and demand parameters follow a particular probability distribution (such as exponential, weibull, cauchy or extreme value). In this paper an MCSTP is considered where availability and demand parameters follow general form of distribution and a generalized equivalent deterministic model (GMCSTP) of MCSTP is obtained. It is also shown that all previous models obtained by different authors can be deduced with the help of GMCSTP. MCSTP with pareto, power function or burr-XII distributions are also considered and equivalent deterministic models are obtained. To illustrate the proposed model two numerical examples are presented and solved using LINGO 13.0 software package.

Keywords: General form of distributions; Multi-choice programming; Stochastic transportation problem; Transformation technique

1 Introduction

The transportation problem is one of the oldest applications of Linear Programming Problem (LPP). The standard form of the transportation problem was first formulated along with the constructive method of solution by Hitchcock (1941). In a classical transportation problem, a product is to be transported from m sources to n destinations. The availability of the product at i^{th} source is denoted by a_i, where $i = 1, 2, \ldots, m$ and the demand required at j^{th} destination is b_j where $j = 1, 2, \ldots, n$. The penalty c_{ij} is the cost coefficient of the objective function which can represent transportation cost, delivery time etc. In many real world situations the availability a_i and demand b_j are not certainly known to Decision Maker (DM). One way to deal such uncertainty is to describe the availability a_i and demand b_j parameters as random variables rather than the deterministic one. These random variables a_i and b_j are assumed to follow a given probability distribution or its probability distribution may be estimated. This type of transportation problem is known as "Stochastic Transportation Problem" (STP). Furthermore, suppose that there exist k routes for transporting

the product from i^{th} source to j^{th} destination and the cost of transporting a unit of product via k^{th} route is denoted by C_{ij}^k. Thus DM have multiple (*i.e.* 'k') route choices for shipping the product from i^{th} source to j^{th} destination and he has to identify exactly one among k routes in such a manner that the combination of choices should minimize the overall transportation cost. With the above discussed objective the STP becomes 'Multi Choice Stochastic Transportation Problem' (MCSTP) in which the cost coefficient C_{ij} are multi-choice and availability a_i and demand b_j are random variables.

MCSTP has been extensively studied by many researchers. Roy et al. (2012) presented an equivalent deterministic model of MCSTP by assuming that both availability a_i and demand b_j as random variables following exponential distribution. Biswal and Samal (2013) obtained an equivalent deterministic model of MCSTP in which they considered that both a_i and b_j follow Cauchy distribution. Mahapatra (2014) also given equivalent deterministic model of MCSTP involving Weibull distribution. Mahapatra et al. (2013) considered the MCSTP involving Extreme value distribution. Barik et al. (2011) presented a stochastic transportation model involving Pareto distribution.

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These random variables a_i and b_j may also be considered to follow Burr-XII or Power Function distributions. Burr-XII may be used in place of normal distribution when data shows some positive skewness. Since a_i and b_j are the physical quantities so it is advisable to use Burr-XII instead of Normal distribution. When upper bound of availability and demand is known, Power Function distribution would be most suitable distribution to fit. In this paper we considered general form of MCSTP, where a_i and b_j are assumed to follow 'General classes of distribution' and obtained a generalised equivalent deterministic model (GMCSTP). All the models discussed above by many authors have been deduced by using the proposed GMCSTP. Three new equivalent deterministic models of MCSTP have also been obtained by considering that both a_i and b_j follow Pareto, Power Function and Burr-XII distribution (only one at a time). An equivalent deterministic GMCSTP has also been obtained by considering that a_i follows any one distribution among Exponential, Weibull, Cauchy, Exterme Value, Pareto, Power Function or Burr-XII and b_j follows any other distribution except that of distribution of a_i. To illustrate the proposed models two numerical examples are taken and solved by using transformation technique given by Biswal and Acharya (2009). Lingo 13.0 software has been used for obtaining the optimal solution.

2 General classes of distributions

Let us consider a random variable y following any of the two general classes of distributions with distribution function (*df*) $F(y)$ as follows:

$$F(y) = 1 - \overline{F}(y) = 1 - \left[ph(y) + q\right]^r, \ y \in (\xi, \phi) \qquad (2.1)$$

and

$$F(y) = 1 - \overline{F}(y) = e^{-ph(y)} \ p \neq 0, \ y \in (\xi, \phi) \qquad (2.2)$$

where $h(y)$ is a monotonic and differentiable function of y and p, q, r and $h(y)$ are chosen such that $F(y)$ in (2.1) and (2.2) are *df* over (ξ, ϕ).

Differentiating (2.1) and (2.2) with respect to y the probability density function (*pdf*), $f(y)$ may be obtained respectively as,

$$f(y) = -prh'(y) \left[ph(y) + q\right]^{r-1} \qquad (2.3)$$

$$f(y) = -ph'(y)e^{-ph(y)} \qquad (2.4)$$

where $F(\xi)=0$ and $F(\phi)=1$.

3 Mathematical model of multi-choice stochastic transportation problem (MCSTP)

In this section a mathematical model of multi-choice transportation problem involving general form of distributions (2.1 or 2.2) is considered. The general form of MCSTP is:

MCSTP1:

$$\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K$$

$$(3.1)$$

subject to,

$$\Pr\left[\sum_{j=1}^{n} x_{ij} \leq a_i\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m \qquad (3.2)$$

$$\Pr\left[\sum_{i=1}^{m} x_{ij} \geq b_j\right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n \qquad (3.3)$$

$$x_{ij} \geq 0 \ \ \forall \ i \text{ and } j \qquad (3.4)$$

where $0 < \alpha_i < 1$, $\forall i$ and $0 < \beta_j < 1$, $\forall j$, are the aspiration levels.

It is assumed that $a_i, \ i = 1, 2, \ldots, m, \ b_j, \ j = 1, 2, \ldots, n$ are random variables following general form of distribution, $\left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\}$ $k = 1, 2, \ldots, K$ are multi-choice parameters and x_{ij} are deterministic decision variables.

The following cases are to be considered:

(i) Only $a_i, \ i = 1, 2, \ldots, m$ follows general form of distribution.

(ii) Only $b_j, \ j = 1, 2, \ldots, n$ follows general form of distribution.

(iii) Both $a_i, \ i = 1, 2, \ldots, m$ and $b_j, \ j = 1, 2, \ldots, n$ follow general form of distribution.

3.1 Only $a_i \ i = 1, 2, \ldots, m$ follows (2.1) or (2.2)

It is considered that $a_i, \ i = 1, 2, \ldots, m$ are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2) consider the probabilistic constraint (3.2),

$$\Pr\left[\sum_{j=1}^{n} x_{ij} \leq a_i\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m$$

or

$$\Pr\left[a_i \geq \sum_{j=1}^{n} x_{ij}\right] \geq 1 - \alpha_i, \quad i = 1, 2, \ldots, m \qquad (3.5)$$

the above inequality (3.5) can be represented as

$$\int_{\sum_{j=1}^{n} x_{ij}}^{\phi_i} f(a_i)da_i \geq 1 - \alpha_i$$

$$\int_{\sum_{j=1}^{n} x_{ij}}^{\phi_i} \frac{d}{da_i} \left[-\overline{F}(a_i) \right] da_i \geq 1 - \alpha_i$$

$$- \overline{F}(a_i)|_{\sum_{j=1}^{n} x_{ij}}^{\phi} \geq 1 - \alpha_i$$

$$- \left[\overline{F}(\phi_i) - \overline{F} \left(\sum_{j=1}^{n} x_{ij} \right) \right] \geq 1 - \alpha_i$$

$$\overline{F} \left(\sum_{j=1}^{n} x_{ij} \right) \geq 1 - \alpha_i$$

$$F \left(\sum_{j=1}^{n} x_{ij} \right) \leq \alpha_i \qquad (3.6)$$

Thus, we obtained a multi-choice deterministic model **MCSTP 2** as follows:

MCSTP 2:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \qquad (3.7)$$

subject to,

$$F \left(\sum_{j=1}^{n} x_{ij} \right) \leq \alpha_i, \quad i = 1, 2, \ldots, m \qquad (3.8)$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad j = 1, 2, \ldots, n \qquad (3.9)$$

$$x_{ij} \geq 0 \quad \forall i \text{ and } j \qquad (3.10)$$

where $\sum_{i=1}^{m} F^{-1}(\alpha_i) \geq \sum_{j=1}^{n} b_j$ (feasibility condition).

3.2 Only $b_j, j = 1, 2, \ldots, n$ follows (2.1) or (2.2)

It is considered that $b_j, i = 1, 2, \ldots, n$ are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2) consider the probabilistic constraint (3.3),

$$\Pr \left[\sum_{i=1}^{m} x_{ij} \geq b_j \right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

or

$$\Pr \left[b_j \leq \sum_{i=1}^{m} x_{ij} \right] \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n \qquad (3.11)$$

the above inequality (3.11) can be represented as

$$\int_{\xi}^{\sum_{i=1}^{m} x_{ij}} f(b_j)db_j \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

$$\int_{\xi}^{\sum_{i=1}^{m} x_{ij}} \frac{d}{db_j} \left[-\overline{F}(b_j) \right] db_j \geq 1 - \beta_j, \quad j = 1, 2, \ldots, n$$

$$- \overline{F}(b_j)|_{\xi}^{\sum_{i=1}^{m} x_{ij}} \geq 1 - \beta_j \quad j = 1, 2, \ldots, n$$

$$- \left[\overline{F} \left(\sum_{j=i}^{m} x_{ij} \right) - 1 \right] \geq 1 - \beta_j \quad j = 1, 2, \ldots, n$$

$$F \left(\sum_{i=1}^{m} x_{ij} \right) \geq 1 - \beta_j \quad j = 1, 2, \ldots, n \qquad (3.12)$$

Thus, we obtained a multi-choice deterministic model **MCSTP 3** as follows:

MCSTP 3:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \qquad (3.13)$$

subject to,

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad i = 1, 2, \ldots, m \qquad (3.14)$$

$$F \left(\sum_{i=1}^{m} x_{ij} \right) \geq 1 - \beta_j \quad j = 1, 2, \ldots, n \qquad (3.15)$$

$$x_{ij} \geq 0 \quad \forall i \text{ and } j \qquad (3.16)$$

where $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} F^{-1}(1 - \beta_j)$ (feasibility condition).

3.3 Both a_i $(i = 1, 2, \ldots, m)$ and b_j $(j = 1, 2, \ldots, n)$ follow (2.1) or (2.2)

It is considered that a_i $(i = 1, 2, \ldots, m)$ and $b_j, j = 1, 2, \ldots, n$ are independent random variable which follows any of two general form of distributions as defined in (2.1) and (2.2).

In view of (3.6) and (3.12) we may obtain a multi-choice deterministic model **GMCSTP** as follows:

GMCSTP:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \qquad (3.17)$$

subject to,

$$F\left(\sum_{j=1}^{n} x_{ij}\right) \le \alpha_i, \quad i = 1, 2, \ldots, m \tag{3.18}$$

$$F\left(\sum_{i=1}^{m} x_{ij}\right) \ge 1 - \beta_j, \quad j = 1, 2, \ldots, n \tag{3.19}$$

$$x_{ij} \ge 0 \quad \forall i \text{ and } j \tag{3.20}$$

where $\sum_{i=1}^{m} F^{-1}(\alpha_i) \ge \sum_{j=1}^{n} F^{-1}(1 - \beta_j)$ (feasibility condition).

4 Different cases of GMCSTP

Consider the following three cases of **GMCSTP**

- (a) when a_i and b_j both follow general form of distribution defined in (2.1).
- (b) when a_i and b_j both follow general form of distribution defined in (2.2).
- (c) when a_i and b_j follow general form of distribution defined in (2.1) and (2.2) respectively or vice-versa.

4.1 When a_i and b_j both follow general form of distribution defined in (2.1)

Let us consider that a_i and b_j follows general form of distribution of the form defined in (2.1) *i.e* $F(y) = 1 - \overline{F}(y) = 1 - [ph(y) + q]^r$, $p \ne 0$, $y \in (\xi, \phi)$.

Putting $F\left(\sum_{j=1}^{n} x_{ij}\right) = 1 - \left[p_i h\left(\sum_{j=1}^{n} x_{ij}\right) + q_i\right]^{r_i}$ in (3.18) of **GMCSTP** and $F\left(\sum_{i=1}^{m} x_{ij}\right) = 1 - \left[p_j' g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j'\right]^{r_j'}$ in (3.19) of **GMCSTP**, we get,

GMCSTP 1:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{4.1}$$

subject to,

$$\left[p_i h\left(\sum_{j=1}^{n} x_{ij}\right) + q_i\right]^{r_i} \ge 1 - \alpha_i, \quad i = 1, 2, \ldots, m \tag{4.2}$$

$$1 - \left[p_j' g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j'\right]^{r_j'} \ge 1 - \beta_j, \quad j = 1, 2, \ldots, n \tag{4.3}$$

$$x_{ij} \ge 0 \quad \forall i \text{ and } j \tag{4.4}$$

4.2 When a_i and b_j both follow general form of distribution defined in (2.2)

Let us consider that a_i and b_j in (2.2) *i.e* $F(y) = 1 - \overline{F}(y) = e^{-ph(y)}$ $p \ne 0$, $y \in (\xi, \phi)$.

Putting $F\left(\sum_{j=1}^{n} x_{ij}\right) = e^{-p_i h\left(\sum_{j=1}^{n} x_{ij}\right)}$ in (3.18) of **GMCSTP** and $F\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-p_j' g\left(\sum_{i=1}^{m} x_{ij}\right)}$ in (3.19) of **GMCSTP**, we get,

GMCSTP 2:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{4.5}$$

subject to,

$$1 - e^{-p_i h\left(\sum_{j=1}^{n} x_{ij}\right)} \ge 1 - \alpha_i, \quad i = 1, 2, \ldots, m \tag{4.6}$$

$$e^{-p_j' g\left(\sum_{i=1}^{m} x_{ij}\right)} \ge 1 - \beta_j, \quad j = 1, 2, \ldots, n \tag{4.7}$$

$$x_{ij} \ge 0 \quad \forall i \text{ and } j \tag{4.8}$$

4.3 When a_i and b_j follow general form of distribution defined in (2.1) and (2.2) respectively or vice-versa

Consider a case when a_i follows any one of general form of distributions defined in (2.1) and (2.2) and b_j follows any one of general form of distribution defined in (2.2) and (2.1) respectively, then in view of **GMCSTP 1** and **GMCSTP 2** we have,

GMCSTP 3:

$$\min: Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{4.9}$$

subject to,

$$\left\{\left[p_i h\left(\sum_{j=1}^{n} x_{ij}\right) + q_i\right]^{r_i}\right\} \text{ or }$$
$$\left\{1 - e^{-p_i h\left(\sum_{j=1}^{n} x_{ij}\right)}\right\} \ge 1 - \alpha_i, \quad i = 1, 2, \ldots, m \tag{4.10}$$

$$\left\{e^{-p_j' g\left(\sum_{i=1}^{m} x_{ij}\right)}\right\} \text{ or }$$
$$\left\{1 - \left[p_j' g\left(\sum_{i=1}^{m} x_{ij}\right) + q_j'\right]^{r_j'}\right\} \ge 1 - \beta_j, \quad j = 1, 2, \ldots, n \tag{4.11}$$

$$x_{ij} \ge 0 \quad \forall i \text{ and } j \tag{4.12}$$

5 Deduction of some previous results along with some new results

In this section we deduce some previous results with the help of GMCSTP 1 and GMCSTP 2. Since GMCSTP 1 and 2 has been modelled with the assumption that both a_i and b_j are random variable. So we are considering only GMCSTP 1 and 2 throughout the paper. One can also consider MCSTP 2 or/and MCSTP 3 according to requirement. Many previous models proposed by Roy et al. (2012), Mahapatra (2014), Biswal and Samal (2013) and Mahapatra et al. (2013) can be deduced from **GMCSTP 1** and **GMCSTP 2** by setting different values of $p_i, p'_j, q_i, q'_j, r_i, r'_j, h\left(\sum_{j=1}^{n} x_{ij}\right)$ and $g\left(\sum_{i=1}^{m} x_{ij}\right)$.

5.1 Deductions using GMCSTP 1 and GMCSTP 2

5.1.1 When a_i and b_j follows exponential distribution

Let us consider that both a_i and b_j follow exponential distribution. In order to deduce the model obtained by S.K. Roy *et al*, we set $p_i = 1, q_i = 0, r_i = \frac{\theta_i}{k_i}, h\left(\sum_{j=1}^{n} x_{ij}\right) = e^{-k_i \sum_{j=1}^{n} x_{ij}}$ and $p'_j = 1, q'_j = 0, r'_j = \frac{\theta'_j}{k'_j}, g\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-k'_j \sum_{i=1}^{m} x_{ij}}$ in **GMCSTP 1** and get,

MCSTP 4:

$$\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.1}$$

subject to,

$$\sum_{j=1}^{n} x_{ij} \leq -\theta_i ln(1 - \alpha_i), \quad i = 1, 2 \ldots m. \tag{5.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq -\theta'_j ln(\beta_j), \quad j = 1, 2 \ldots n. \tag{5.3}$$

$$x_{ij} \geq 0 \quad \forall \text{ i and j} \tag{5.4}$$

where $\sum_{i=1}^{m} \{-\theta_i ln(1 - \alpha_i)\} \geq \sum_{j=1}^{n} \{-\theta'_j ln(\beta_j)\}$ (feasibility condition) and $a_i \geq 0, b_j \geq 0$ and $\{\theta_i, \theta'_j\} > 0$ are the parameters of exponential distribution. The above **MCSTP 4** is same as obtained by Roy et al. (2012).

5.1.2 When a_i and b_j follows Weibull distribution

Mahapatra (2014) presented a model by considering both a_i and b_j follow weibull distribution which can be obtained by setting $p_i = 1, q_i = 0, r_i = \frac{\delta_i^{-\gamma_i}}{k_i}, h\left(\sum_{j=1}^{n} x_{ij}\right) = e^{-k_i \left(\sum_{j=1}^{n} x_{ij}\right)^{\gamma_i}}$ and $p'_j = 1, q'_j = 0, r'_j = \frac{\delta_j'^{-\gamma'_j}}{k'_j}, g\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-k'_j \left(\sum_{i=1}^{m} x_{ij}\right)^{\gamma'_j}}$ in **GMCSTP 1**, as follows:

MCSTP 5:

$$\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.5}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq e^{\left[ln\delta_i + \frac{1}{\gamma_i} ln\{-ln(1-\alpha_i)\} \right]} \quad i = 1, 2 \ldots m. \tag{5.6}$$

$$\sum_{i=1}^{m} x_{ij} \geq e^{\left[ln\delta'_j + \frac{1}{\gamma'_j} ln\{-ln(\beta_j)\} \right]} \quad j = 1, 2 \ldots n. \tag{5.7}$$

$$x_{ij} \geq 0 \quad \forall \text{ i and j} \tag{5.8}$$

where $\sum_{i=1}^{m} e^{\left[ln\delta_i + \frac{1}{\gamma_i} ln\{-ln(1-\alpha_i)\} \right]} \geq \sum_{j=1}^{n} e^{\left[ln\delta'_j + \frac{1}{\gamma'_j} ln\{-ln(\beta_j)\} \right]}$ (feasibility condition) and $a_i \geq 0, b_j \geq 0$ and $\{\gamma_i, \gamma'_j\} > 0$ and $\{\delta_i, \delta'_j\} > 0$ are shape and scale parameters.

The above **MCSTP 5** is same as obtained by Mahapatra (2014).

5.1.3 When a_i and b_j follows Cauchy distribution

Biswal and Samal (2013) proposed MCSTP model by considering that a_i and b_j follow Cauchy distribution. On Setting $p_i = -\frac{1}{\pi}, q_i = \frac{1}{2}, r_i = 1, h\left(\sum_{j=1}^{n} x_{ij}\right) = \tan^{-1} \frac{\sum_{j=1}^{n} x_{ij} - l_{a_i}}{s_{a_i}}$ and $p'_j = -\frac{1}{\pi}, q'_j = \frac{1}{2}, r'_j = 1, g\left(\sum_{i=1}^{m} x_{ij}\right) = \tan^{-1} \frac{\sum_{i=1}^{m} x_{ij} - l_{b_j}}{s_{b_j}}$ in **GMCSTP 1**, we get,

MCSTP 6:

$$\text{min: } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \tag{5.9}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq l_{a_i} + s_{a_i} tan\left(\pi\alpha_i - \frac{\pi}{2}\right), \quad i = 1, 2, \ldots, m \tag{5.10}$$

$$\sum_{i=1}^{m} x_{ij} \geq l_{b_j} + s_{b_j} tan\left(\frac{\pi}{2} - \pi\beta_j\right), \quad j = 1, 2, \ldots, n \tag{5.11}$$

$$x_{ij} \geq 0 \quad \forall \text{ i and j} \tag{5.12}$$

where $\sum_{i=1}^{m} l_{a_i} + s_{a_i} tan\left(\pi\alpha_i - \frac{\pi}{2}\right) \geq \sum_{j=1}^{n} l_{b_j} + s_{b_j} tan\left(\frac{\pi}{2} - \pi\beta_j\right)$ (feasibility condition) and $-\infty < a_i < +\infty$, $-\infty < b_j < +\infty$ and $l_{a_i}, l_{b_j} > 0$ and $s_{a_i}, s_{b_j} > 0$ are the location and scale parameter of a_i and b_j, respectively, Which is a multi-choice approach of the model proposed by Biswal and Samal (2013).

5.1.4 When a_i and b_j follows extreme value distribution

Setting $p = 1, h\left(\sum_{j=1}^{n} x_{ij}\right) = e^{-\frac{\left(\sum_{j=1}^{n} x_{ij} - \gamma_i\right)}{\delta_i}}$ and $p' = 1, g\left(\sum_{i=1}^{m} x_{ij}\right) = e^{-\frac{\left(\sum_{i=1}^{m} x_{ij} - \gamma_j'\right)}{\delta_j'}}$ in **GMCSTP 2** it deduces to,

MCSTP 7:

$$\text{min:} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \quad (5.13)$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq \gamma_i - \delta_i \left[ln\{-ln(\alpha_i)\} \right], \quad i = 1, 2, \ldots, m \quad (5.14)$$

$$\sum_{i=1}^{m} x_{ij} \geq \gamma_j' - \delta_j' \left[ln\{-ln(\beta_j)\} \right], \quad j = 1, 2, \ldots, n \quad (5.15)$$

$$x_{ij} \geq 0 \quad \forall i \text{ and } j \quad (5.16)$$

where $\sum_{i=1}^{m} \left[\gamma_i - \delta_i \left[ln\{-ln(\alpha_i)\} \right] \right] \geq \sum_{j=1}^{n} \left[\gamma_j' - \delta_j' \left[ln\{-ln(\beta_j)\} \right] \right]$ (feasibility condition) and $-\infty < a_i < +\infty, -\infty < b_j < +\infty$ and $\{\gamma_i, \gamma_j'\} > 0$ and $\{\delta_i, \delta_j'\} > 0$ are location and scale parameters of extreme value distribution, which is same as obtained by Mahapatra et al. (2013).

5.2 Some new results using GMCSTP 1, GMCSTP 2 and GMCSTP 3

5.2.1 When a_i and b_j follow Pareto distribution

Let us consider the MCSTP in which a_i and b_j follow Pareto distribution. By setting $p_i = d_i^{-k_i}, q_i = 0, r_i = -\frac{\theta_i}{k_i}, h\left(\sum_{j=1}^{n} x_{ij}\right) = \left(\sum_{j=1}^{n} x_{ij}\right)^{k_i}$ and $p_j' = d_j^{-k_j'}, q_j' = 0, r_i' = -\frac{\theta_j'}{k_j'}, g\left(\sum_{j=1}^{n} x_{ij}\right) = \left(\sum_{j=1}^{n} x_{ij}\right)^{k_j'}$ in **GMCSTP 1**, we get,

MCSTP 8:

$$\text{min:} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \quad (5.17)$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq \frac{d_i}{(1-\alpha_i)^{\frac{1}{\theta_i}}}, \quad i = 1, 2, \ldots, m \quad (5.18)$$

$$\sum_{i=1}^{m} x_{ij} \geq \frac{d_j'}{(\beta_j)^{\frac{1}{\theta_j'}}}, \quad j = 1, 2, \ldots, n \quad (5.19)$$

$$x_{ij} \geq 0 \quad \forall i \text{ and } j \quad (5.20)$$

where $\sum_{i=1}^{m} \left(\frac{d_i}{(1-\alpha_i)^{\frac{1}{\theta_i}}} \right) \geq \sum_{j=1}^{n} \left(\frac{d_j'}{(\beta_j)^{\frac{1}{\theta_j'}}} \right)$ (feasibility condition) and $\{d_i, d_j'\} > 0$ and $\{\theta_i, \theta_j'\} > 0$ are scale and shape parameters respectively and $a_i \geq d_i$ and $b_j \geq d_j'$.

5.2.2 When a_i and b_j follow Burr-XII distribution

Setting $p = \theta_i, q = 1, r = -k_i, h\left(\sum_{j=1}^{n} x_{ij}\right) = \left(\sum_{j=1}^{n} x_{ij}\right)^{\delta_i}$ and $p' = \theta_j', q' = 1, r' = -k_j', g\left(\sum_{i=1}^{m} x_{ij}\right) = \left(\sum_{i=1}^{m} x_{ij}\right)^{\delta_j'}$ in **GMCSTP 1** we get,

MCSTP 9:

$$\text{min:} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K$$

$$(5.21)$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq \left[\frac{(1-\alpha_i)^{-\frac{1}{k_i}} - 1}{\theta_i} \right]^{\frac{1}{\delta_i}}, \quad i = 1, 2, \ldots, m \quad (5.22)$$

$$\sum_{i=1}^{m} x_{ij} \geq \left[\frac{\beta_j^{-\frac{1}{k_j'}} - 1}{\theta_j'} \right]^{\frac{1}{\delta_j'}}, \quad j = 1, 2, \ldots, n \quad (5.23)$$

$$x_{ij} \geq 0 \quad \forall \text{ i and j} \quad (5.24)$$

where $\sum_{i=1}^{m} \left[\frac{(1-\alpha_i)^{-\frac{1}{k_i}} - 1}{\theta_i} \right]^{\frac{1}{\delta_i}} \geq \sum_{j=1}^{n} \left[\frac{\beta_j^{-\frac{1}{k_j'}} - 1}{\theta_j'} \right]^{\frac{1}{\delta_j'}}$ (feasibility condition) and $a_i \geq 0, b_j \geq 0$ and $\{\theta_i, \theta_j'\} > 0$ and $\{k_i, k_j'\} > 0$ are shape parameters of Burr-XII distribution.

5.2.3 When a_i and b_j follow power function distribution

Setting $p_i = -\theta_i, h\left(\sum_{j=1}^{n} x_{ij}\right) = ln\left(\frac{\sum_{j=1}^{n} x_{ij}}{d_i}\right)$ and $p_j = -\theta_j', h\left(\sum_{i=1}^{m} x_{ij}\right) = ln\left(\frac{\sum_{i=1}^{m} x_{ij}}{d_j'}\right)$ in **GMCSTP 2**, we get,

MCSTP 10:

$$\text{min:} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k \right\} x_{ij}, \quad k = 1, 2, \ldots, K \quad (5.25)$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq d_i \alpha_i^{\frac{1}{\theta_i}}, \quad i = 1, 2, \ldots, m \quad (5.26)$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j'(1-\beta_j)^{\frac{1}{\theta_j'}}, \quad j = 1, 2, \ldots, n \quad (5.27)$$

$$x_{ij} \geq 0 \quad \forall i \text{ and } j \quad (5.28)$$

where $\sum_{i=1}^{m}\left(d_i\alpha_i^{\frac{1}{\theta_i}}\right) \geq \sum_{j=1}^{n}\left(d_j'(1-\beta_j)^{\frac{1}{\theta_j'}}\right)$ (feasibility condition) and $\{d_i, d_j'\} > 0$ and $\{\theta_i, \theta_j'\} > 0$ are the scale and shape parameters of $a_i \geq 0$ and $b_j \geq 0$ respectively.

5.2.4 When a_i follows Burr XII distribution and b_j follows Extreme value distribution

Setting $p_i = \theta_i, q_i = 1, r_i = -k_i, h\left(\sum_{j=1}^{n}x_{ij}\right) = \left(\sum_{j=1}^{n}x_{ij}\right)^{\delta_i}$ and $p_j' = 1, g\left(\sum_{i=1}^{m}x_{ij}\right) = e^{-\frac{\left(\sum_{i=1}^{m}x_{ij}-\gamma_j'\right)}{\delta_j'}}$ in **GMCSTP 3** we get,

MCSTP 11:

$$\text{min}: Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\}x_{ij}, \ k = 1, 2, \ldots, K \quad (5.29)$$

Subject to:

$$\sum_{j=1}^{n}x_{ij} \leq \left[\frac{(1-\alpha_i)^{-\frac{1}{k_i}}-1}{\theta_i}\right]^{\frac{1}{\delta_i}}, \ i = 1, 2, \ldots, m \quad (5.30)$$

$$\sum_{i=1}^{m}x_{ij} \geq \gamma_j' - \delta_j'[\,ln\{-ln(\beta_j)\}]\,, \ j = 1, 2, \ldots, n \quad (5.31)$$

$$x_{ij} \geq 0 \quad \forall \, i \text{ and } j \quad (5.32)$$

where $\sum_{i=1}^{m}\left[\frac{(1-\alpha_i)^{-\frac{1}{k_i}}-1}{\theta_i}\right]^{\frac{1}{\delta_i}} \geq \sum_{j=1}^{n}\left[\gamma_j' - \delta_j'[\,ln\{-ln(\beta_j)\}]\right]$.

5.2.5 When a_i follows power function distribution and b_j follows Pareto distribution

Setting $p_i = -\theta_i, \ h\left(\sum_{i=1}^{m}x_{ij}\right) = ln\left(\frac{\sum_{j=1}^{n}x_{ij}}{d_i}\right)$ and $p_j' = d_j'^{-k_j'}, q_j' = 0, r_i = -\frac{\theta_j'}{k_j'}, \ g\left(\sum_{j=1}^{n}x_{ij}\right) = \left(\sum_{j=1}^{n}x_{ij}\right)^{k_j'}$ in **GMCSTP 3**.

MCSTP 12:

$$\text{min}: Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\}x_{ij}, \ k = 1, 2, \ldots, K \quad (5.33)$$

subject to:

$$\sum_{j=1}^{n}x_{ij} \leq d_i\alpha_i^{\frac{1}{\theta_i}}, \ i = 1, 2, \ldots, m \quad (5.34)$$

$$\sum_{i=1}^{m}x_{ij} \geq \frac{d_j'}{(\beta_j)^{\frac{1}{\theta_j'}}}, \ j = 1, 2, \ldots, n \quad (5.35)$$

$$x_{ij} \geq 0 \quad \forall \, i \text{ and } j \quad (5.36)$$

where $\sum_{i=1}^{m}\left(d_i\alpha_i^{\frac{1}{\theta_i}}\right) \geq \sum_{j=1}^{n}\left(\frac{d_j'}{(\beta_j)^{\frac{1}{\theta_j'}}}\right)$ (feasibility condition).

6 Numerical illustrations

We consider the numerical example taken by (Mahapatra et al. 2013). Data for multi-choice cost C_{ij}^k are appended below in Table 1.

6.1 Illustration 1

Let us consider that we have three known parameters of availability a_1, a_2, a_3 follow Burr-XII distribution. The specified probability levels and shape parameters of a_1, a_2, a_3 are given in Table 2.

Further, consider that we have four known parameters of demand b_1, b_2, b_3, b_4 follow extreme value distribution. The specified probability levels and location and scale parameters of b_1, b_2, b_3, b_4 are given in Table 3.

Using the data provided in Tables 1, 2 and 3 the following equivalent multi-choice deterministic transportation problem is formulated with the help of GMCSTP 3 as:

$$\begin{aligned}\text{min}:z = &\{10, 11, 12\}x_{11} + \{15, 16\}x_{12} + \{21, 22, 23, 24\}x_{13} \\ &+ \{21, 23, 25\}x_{14} + \{15, 17, 19, 21, 23, 25\}x_{21} \\ &+ \{10, 12, 14, 16, 18, 20\}x_{22} + \{9, 10, 11\}x_{23} \\ &+ \{18, 19\}x_{24} + \{20, 21, 22, 23, 24, 25, 26\}x_{31} \\ &+ \{10, 11, 12, 13, 14, 16, 17\}x_{32} \\ &+ \{20, 22, 25\}x_{33} + \{15, 20\}x_{34}\end{aligned}$$

subject to,

$$\sum_{j=1}^{4}x_{1j} \leq 967.544404 \quad (6.1)$$

$$\sum_{j=1}^{4}x_{2j} \leq 762.934875 \quad (6.2)$$

$$\sum_{j=1}^{4}x_{3j} \leq 612.817850 \quad (6.3)$$

Table 1 Multi-choice transportation cost for route x_{ij}

Sl. no.	Route: x_{ij}	Transportation cost(in Rupees) C_{ij}^k: per unit (1 unit = 10 kg)
1	$(1, 1): x_{11}$	10 or 11 or 12
2	$(1, 2): x_{12}$	15 or 16
3	$(1, 3): x_{13}$	21 or 22 or 23 or 24
4	$(1, 4): x_{14}$	21 or 23 or 25
5	$(2, 1): x_{21}$	15 or 17 or 19 or 21 or 23 or 25
6	$(2, 2): x_{22}$	10 or 12 or 14 or 16 or 18 or 20
7	$(2, 3): x_{23}$	9 or 10 or 11
8	$(2, 4): x_{24}$	18 or 19
9	$(3, 1): x_{31}$	20 or 21 or 22 or 23 or 24 or 25 or 26
10	$(3, 2): x_{32}$	10 or 11 or 12 or 13 or 14 or 15 or 16 or 17
11	$(3, 3): x_{33}$	20 or 22 or 25
12	$(3, 4): x_{34}$	15 or 20

Table 2 Specified probability levels and shape parameters of a_i

Random parameters a_i	Specified probability levels	Shape parameters 1	Shape parameters 2
a_1	0.01	0.002	0.73
a_2	0.02	0.004	0.76
a_3	0.03	0.006	0.79

$$\sum_{i=1}^{3} x_{i1} \geq 615.992671 \tag{6.4}$$

$$\sum_{i=1}^{3} x_{i2} \geq 511.880781 \tag{6.5}$$

$$\sum_{i=1}^{3} x_{i3} \geq 408.347897 \tag{6.6}$$

$$\sum_{i=1}^{3} x_{i4} \geq 305246388 \tag{6.7}$$

$$x_{ij} \geq 0, i = 1, 2, 3; \; j = 1, 2, 3, 4.$$

Now using the transformation technique proposed by Biswal and Acharya (2009), we obtain the following multi-choice deterministic transportation problem:

$$\text{min: } z = t_{11}x_{11} + t_{12}x_{12} + t_{13}x_{13} + t_{14}x_{14}$$
$$+ t_{21}x_{21} + t_{22}x_{22} + t_{23}x_{23} + t_{24}x_{24}$$
$$+ t_{31}x_{31} + t_{32}x_{32} + t_{33}x_{33} + t_{34}x_{34}$$

subject to, (6.1)-(6.7)

where,

$$t_{11} = 10z_{11}^1 z_{11}^2 + 11z_{11}^1 \left(1 - z_{11}^2\right) + 12 \left(1 - z_{11}^1\right) z_{11}^2$$

$$t_{12} = 15z_{12}^1 + 16 \left(1 - z_{12}^1\right)$$

$$t_{13} = 21z_{13}^1 z_{13}^2 + 22z_{13}^1 \left(1 - z_{13}^2\right) + 23 \left(1 - z_{13}^1\right) z_{13}^2$$
$$+ 24 \left(1 - z_{13}^1\right) \left(1 - z_{13}^2\right)$$

$$t_{14} = 21z_{14}^1 z_{14}^2 + 23z_{14}^1 (1 - z_{14}^2 + 25 \left(1 - z_{14}^1\right) z_{14}^2$$

Table 3 Specified probability levels, location and scale parameters of b_j

Random parameters b_j	Specified probability levels	Location parameters	Scale parameters
b_1	0.04	600	5
b_2	0.05	500	4
b_3	0.06	400	3
b_4	0.07	300	2

$$t_{21} = 15z_{21}^1 \left(1 - z_{21}^2\right) \left(1 - z_{21}^3\right)$$
$$+ 17 \left(1 - z_{21}^1\right) z_{21}^2 \left(1 - z_{21}^3\right)$$
$$+ 19 \left(1 - z_{21}^1\right) \left(1 - z_{21}^2\right) z_{21}^3$$
$$+ 21z_{21}^1 z_{21}^2 \left(1 - z_{21}^3\right) + 23 \left(1 - z_{21}^1\right) z_{21}^2 z_{21}^3$$
$$+ 25z_{21}^1 \left(1 - z_{21}^2\right) z_{21}^3$$

$$t_{22} = 10z_{22}^1 \left(1 - z_{22}^2\right) \left(1 - z_{22}^3\right)$$
$$+ 12 \left(1 - z_{22}^1\right) z_{22}^2 \left(1 - z_{22}^3\right)$$
$$+ 14z_{22}^1 z_{22}^2 \left(1 - z_{22}^3\right)$$
$$+ 16 \left(1 - z_{22}^1\right) \left(1 - z_{22}^2\right) z_{22}^3$$
$$+ 18z_{22}^1 \left(1 - z_{22}^2\right) z_{22}^3 + 20 \left(1 - z_{22}^1\right) z_{22}^2 z_{22}^3$$

$$t_{23} = 9z_{23}^1 z_{23}^2 + 10z_{23}^1 \left(1 - z_{23}^2\right) + 11 \left(1 - z_{23}^1\right) z_{23}^2$$

$$t_{24} = 18z_{24}^1 + 19 \left(1 - z_{24}^1\right)$$

$$t_{31} = 20 \left(1 - z_{31}^1\right) \left(1 - z_{31}^2\right) \left(1 - z_{31}^3\right)$$
$$+ 21z_{31}^1 \left(1 - z_{31}^2\right) \left(1 - z_{31}^3\right)$$
$$+ 22 \left(1 - z_{31}^1\right) z_{31}^2 \left(1 - z_{31}^3\right)$$
$$+ 23 \left(1 - z_{31}^1\right) \left(1 - z_{31}^2\right) \left(1 - z_{31}^3\right)$$
$$+ 24z_{31}^1 z_{31}^2 \left(1 - z_{31}^3\right) + 25z_{31}^1 \left(1 - z_{31}^2\right) z_{31}^3$$
$$+ 26 \left(1 - z_{31}^1\right) z_{31}^2 z_{31}^3$$

$$t_{32} = 10z_{32}^1 z_{32}^2 z_{32}^3) + 11 \left(1 - z_{32}^1\right) z_{32}^2 z_{32}^3$$
$$+ 12z_{32}^1 \left(1 - z_{32}^2\right) z_{32}^3 + 13z_{32}^1 z_{32}^2) \left(1 - z_{32}^3\right)$$
$$+ 14 \left(1 - z_{32}^1\right) \left(1 - z_{32}^2\right) z_{32}^3$$
$$+ 15z_{32}^1 \left(1 - z_{32}^2\right) \left(1 - z_{32}^3\right)$$
$$+ 16 \left(1 - z_{32}^1\right) z_{32}^2 \left(1 - z_{32}^3\right)$$
$$+ 17 \left(1 - z_{32}^1\right) \left(1 - z_{32}^2\right) \left(1 - z_{32}^3\right)$$

$$t_{33} = 20z_{33}^1 z_{33}^2 + 22z_{33}^1 \left(1 - z_{23}^2\right) + 25 \left(1 - z_{33}^1\right) z_{33}^2$$

$$t_{34} = 15z_{34}^1 + 20 \left(1 - z_{34}^1\right)$$

$$1 \leq z_{11}^1 + z_{11}^2 \leq 2$$
$$1 \leq z_{14}^1 + z_{14}^2 \leq 2$$
$$1 \leq z_{21}^1 + z_{21}^2 + z_{21}^3 \leq 2$$
$$1 \leq z_{22}^1 + z_{22}^2 + z_{22}^3 \leq 2$$
$$1 \leq z_{23}^1 + z_{23}^2 \leq 2$$
$$1 \leq z_{31}^1 + z_{31}^2 + z_{31}^3 \leq 2$$
$$1 \leq z_{33}^1 + z_{33}^2 \leq 2$$

where, $x_{ij} \geq 0, i = 1, 2, 3; \; j = 1, 2, 3, 4.$

The above non-linear mixed integer programming problem is solved by using LINGO 13.0 software package and the optimal solution is obtained as: $x_{11} = 615.9927, x_{22} = 382.1037, x_{23} = 408.3469, x_{32} = 129.7771, x_{34} = 305.2464$ and rest of the x_{ij} are zero. The minimum

Table 4 Specified probability levels, scale and shape parameters of a_i

Random parameters a_i	Specified probability levels	Scale parameters	Shape parameters
a_1	0.01	1000	100
a_2	0.02	800	70
a_3	0.03	700	60

Table 5 Specified probability levels, scale and shape parameters of b_j

Random parameters b_j	Specified probability levels	Scale parameters	Shape parameters
b_1	0.04	350	5
b_2	0.05	300	6
b_3	0.06	270	7
b_4	0.07	230	8

transportation cost is 19532.56 obtained by choosing multi-choice cost as follows:

x_{ij} : x_{11} x_{12} x_{13} x_{14} x_{21} x_{22} x_{23} x_{24} x_{31} x_{32} x_{33} x_{34}
value of C_{ij}^k : 10   15   21   23   15   10   9   18   22   10   22   15

6.2   Illustration 2

Again consider, in the above **illustration 1** the availability a_1, a_2, a_3 are supposed to follow Power Function distribution and the demand b_1, b_2, b_3, b_4 are assumed to follow Pareto distribution. The specified probability levels, scale and shape parameters of a_1, a_2, a_3 are given in Table 4 and of b_1, b_2, b_3, b_4 are given in Table 5 respectively.

Solving this in the similar manner optimal solutions are obtained as $x_{11} = 666.2789$, $x_{22} = 352.9524$, $x_{23} = 403.5651$, $x_{32} = 141.3123$, $x_{34} = 320.6938$ and rest of the x_{ij} are zero. The minimum transportation cost is 20047.93 obtained by choosing multi-choice cost as follows:

x_{ij} : x_{11} x_{12} x_{13} x_{14} x_{21} x_{22} x_{23} x_{24} x_{31} x_{32} x_{33} x_{34}
value of C_{ij}^k : 10   15   23   21   21   10   9   18   25   10   20   15

7   Conclusion

In this paper we have considered a MCSTP where cost coefficient of objective function are assumed to be of multi-choice type and random availability and demand of product are assumed to follow general form of distributions. With this generalized formulation of MCSTP the DM becomes capable to fit any distribution among exponential, weibull, cauchy, extreme value, power function, burr-XII and pareto according to the nature of data. Thus the present model can be applied in several situations of transportation problems when demand and availability are restricted to follow a particular probability distribution. Here, only upto eight choices of multi-choice

cost parameters are considered because we were much concerned about random parameters. In further studies extended multi-choice parameters may also be taken into account.

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