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# Certain class of higher-dimensional simplicial complexes and universal $C^*$-algebras

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## Abstract

In this article we introduce a universal $C^*$-algebras associated to certain simplicial flag complexes. We denote it by $\mathcal{C}_{\Gamma^n}$ it is a subalgebra of the noncommutative $n$-sphere which introduced by J.Cuntz. We present a technical lemma to determine the quotient of the skeleton filtration of a general universal $C^*$-algebra associated to a simplicial flag complex. We examine the $K$-theory of this algebra. Moreover we prove that any such algebra divided by the ideal $I_2$ is commutative.

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## Introduction

In this section, we give a survey of some basic definitions and properties of the universal $C^*$-algebra associated to a certain flag complex which we will use in the sequel. Such algebras in general was introduced first by Cuntz (2002) and studied by Omran (2005, 2013).

**Definition 1.** *A simplicial complex $\Sigma$ consists of a set of vertices $V_\Sigma$ and a set of non-empty subsets of $V_\Sigma$, the simplexes in $\Sigma$, such that:*

- *If $s \in V_\Sigma$, then $\{s\} \in \Sigma$.*
- *If $F \in \Sigma$ and $\emptyset \neq E \subset F$ then $E \in \Sigma$.*

*A simplicial complex $\Sigma$ is called flag or full, if it is determined by its 1-simplexes in the sense that $\{s_0, \dots, s_n\} \in \Sigma \iff \{s_i, s_j\} \in \Sigma$ for all $0 \le i < j \le n$.*

$\Sigma$ is called locally finite if every vertex of $\Sigma$ is contained in only finitely many simplexes of $\Sigma$, and finite-dimensional (of dimension $\leqslant n$) if it contains no simplexes with more than $n+1$-vertices. For a simplicial complex $\Sigma$ one can define the topological space $|\Sigma|$ associated to this complex. It is called the “geometric realization” of the complex and can be defined as the space of maps $f : V_\Sigma \longrightarrow [0, 1]$ such that $\sum_{s \in V_\Sigma} f(s) = 1$ and $f(s_0)$..... $f(s_i) = 0$ whenever $\{s_0, \dots, s_i\} \notin \Sigma$. If $\Sigma$ is locally finite, then $|\Sigma|$ is locally compact.

Let $\Sigma$ be a locally finite flag simplicial complex. Denote by $V_\Sigma$ the set of its vertices. Define $\mathcal{C}_\Sigma$ as the universal $C^*$-algebra with positive generators $h_s, s \in V$, satisfying the relations

$$h_{s_0} h_{s_1} \dots h_{s_n} = 0 \text{ whenever } \{s_0, s_1, \dots, s_n\} \notin V_\Sigma,$$

$$\sum_{s \in V_\Sigma} h_s h_t = h_t \quad \forall\, t \in V_\Sigma.$$

Here the sum is finite, because $\Sigma$ is locally finite.

$\mathcal{C}^{ab}_\Sigma$ is the abelian version of the universal $C^*$-algebra above, i.e. satisfying in addition $h_s h_t = h_t h_s$ forall $s, t \in V_\Sigma$. Denote by $I_k$ the ideal in $\mathcal{C}_\Sigma$ generated by products containing at least $n+1$ different generators. The filtration $(I_k)$ of $\mathcal{C}_\Sigma$ is called the skeleton filtration.

Let

$$\Delta := \left\{ (s_0, \dots, s_n) \in \mathbb{R}^{n+1} \mid 0 \le s_i \le 1, \sum_{i=1}^{n} s_i = 1 \right\}$$

be the standard $n$-simplex. Denote by $\mathcal{C}_\Delta$ the associated universal $C^*$-algebra with generators $h_s$, $s \in \{s_0, \dots, s_n\}$, such that $h_s \ge 0$ and $\sum_s h_s = 1$. Denote by $\mathcal{I}_\Delta$ the ideal in $\mathcal{C}_\Delta$ generated by products of generators containing all the $h_{s_i}$, $i = 0, \dots, n$. For each $k$, denote by $I_k$ the ideal in $\mathcal{C}_\Delta$ generated by all products of generators $h_s$ containing at least $k+1$ pairwise different generators. We also denote by $I^{ab}_k$ the image of $I_k$ in $\mathcal{C}^{ab}_\Delta$. The algebra $\mathcal{C}_\Delta$ and their

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$K$-Theory was studied in details in (Omran and Gouda 2012). For any vertex $t$ in $\Delta$ there is a natural evaluation map $\mathcal{C}_\triangle \longrightarrow \mathbb{C}$ mapping the generators $h_t$ to 1 and all the other generators to 0. The following propositions are due to Cuntz (2002).

**Proposition 1.** (*i*) *The evaluation map* $\mathcal{C}_\triangle \longrightarrow \mathbb{C}$ *defined above induces an isomorphism in K-theory.* (*ii*) *The surjective map* $\mathcal{I}_\triangle \longrightarrow \mathcal{I}_\triangle^{ab}$ *induces an isomorphism in K-theory, where* $\mathcal{I}_\triangle^{ab}$ *is the abelianization of* $\mathcal{I}_\triangle$.

We can observe that $I_k$ is the kernel of the evaluation map which define above so we can conclude that $I_k$ is closed.

**Remark 1.** *Let* $\Delta$ *and* $\mathcal{I}_\triangle \subset \mathcal{C}_\triangle$ *as above. Then* $K_*(\mathcal{I}_\triangle) \cong K_*(\mathbb{C}),\ * = 0, 1,$ *if the dimension n of* $\triangle$ *is even and* $K_*(\mathcal{I}_\triangle) \cong K_*(C_0(0,1)),\ * = 0, 1,$ *if the dimension n of* $\triangle$ *is odd.*

**Proposition 2.** *Let* $\Sigma$ *be a locally finite simplicial complex. Then* $\mathcal{C}_\Sigma^{ab}$ *is isomorphic to* $C_0(|\Sigma|)$*, the algebra of continuous functions vanishing at infinity on the geometric realization* $|\Sigma|$ *of* $\Sigma$.

## Universal $C^*$-algebras associated to certain complexes

Universal $C^*$-algebras is a $C^*$-algebras generated by generators and relations. Many $C^*$-algebras can be constructed in the form of universal $C^*$-algebras an important example for universal $C^*$-algebras is Cuntz algebras $O_n$ the existence of this algebras and their $K$-theory was introduced by Cuntz (1981, 1984) more other examples of universal $C^*$-algebras can be found in (Cuntz 1993; Davidson 1996). In the following, we introduce a general technical lemma to compute the quotient of the skeleton filtration for a general algebra associated to simplicial complex.

For a subset $W \subset V_\Sigma$, let $\Gamma \subset \Sigma$ be the subcomplex generated by $W$ and let $\mathcal{I}_\Gamma$ be the ideal in $\mathcal{C}_\Gamma$ generated by products containing all generators of $\mathcal{C}_\Gamma$.

**Lemma 1.** *Let* $\mathcal{C}_\Sigma$ *and* $\mathcal{C}_\Gamma$ *as above, then we have*

$$I_k/I_{k+1} \cong \bigoplus\nolimits_{W \subset V_\Sigma, |W|=k+1} \mathcal{I}_\Gamma$$

*Proof.* $\mathcal{C}_\Sigma/I_{k+1}$ is generated by the images $\dot{h}_i,\ i \in V_\Sigma$ of the generators in the quotient.

Given a subset $W \subset V_\Sigma$ with $|W| = k+1$, let

$$\mathcal{C}_{\Gamma'} = C^*(\{\dot{h}_i | i \in W\}) \subset \mathcal{C}_\Sigma/I_{k+1}.$$

Let $\mathcal{I}_{\Gamma'}$ denote the ideal in $\mathcal{C}_{\Gamma'}$ generated by products containing all generators $\dot{h}_i,\ i \in \Gamma'$, and let $\mathcal{B}_\Gamma$ denote its closure. If $W \neq W'$, then $\mathcal{B}_\Gamma \mathcal{B}_{\Gamma'} = 0$, because the product of any two elements in $\mathcal{B}_\Gamma$ and $\mathcal{B}_{\Gamma'}$ contains products of more than $k+1$-different generators, which are equal to zero in the algebra $\mathcal{C}_\Sigma/I_{k+1}$

It is clear that $\mathcal{B}_\Gamma \subset I_k/I_{k+1}$ so that

$$\bigoplus\nolimits_{W \subset V_\Sigma, |W|=k+1} \mathcal{B}_\Gamma \subset I_k/I_{k+1}.$$

Conversely, let $x \in I_k/I_{k+1}$. Then there is a sequence $(x_n)$ converging to $x$, such that each $x_n$ is a sum of monomials $m_s$ in $\dot{h}_i$ containing at least $k+1$-different generators. Then $m_s \in \mathcal{B}_\Gamma$ for some $W$ and

$$x_n = \sum m_s \in \bigoplus\nolimits_{W \subset V_\Sigma, |W|=k+1} \mathcal{B}_\Gamma.$$

The space $\bigoplus_{W \subset V, |W|=k+1} \mathcal{B}_\Gamma$ is closed, because it is a direct sum of closed ideals. It follows that

$$I_k/I_{k+1} = \bigoplus\nolimits_{W \subset V_\Sigma, |W|=k+1} \mathcal{B}_\Gamma$$

Let now

$$\pi_W : \mathcal{C}_\Sigma \longrightarrow \mathcal{C}_\Gamma.$$

be the canonical evaluation map defined by

$$\pi_W(h_i) = \begin{cases} h_i^{'} & \forall i \in W \\ 0 & \text{if } i \notin W, \end{cases}$$

where $h_i^{'}$ denotes the generator in $\mathcal{C}_\Gamma$ corresponding to the index $i$ in $W$, in other words

$$\mathcal{C}_\Gamma = C^*(h_i^{'} | i \in W).$$

We prove that $\pi_W(I_{k+1}) = 0$. Since polynomials of the form

$$\sum \ldots h_{i_0} \ldots h_{i_j} \ldots h_{i_{k+1}} \ldots, \quad i_0, \ldots, i_j, \ldots, i_{k+1}, \ldots \in V_\Sigma$$

are dense in $I_{k+1}$ , it is enough to show that $\pi_W(x) = 0$ for each such polynomial $x$. We have

$$\pi_W(x) = \sum \ldots h_{i_0}^{'} \ldots h_{i_j}^{'} \ldots h_{i_{k+1}}^{'} \ldots = 0,$$

since there is at least one $i_l$ which is not in $W$. For this index $\pi_W(h_{i_l}) = 0$. Thus $\pi_W(x) = 0$. Therefore $\pi_W$ descends to a homomorphism

$$\dot{\pi_W} : \mathcal{C}_\Sigma/I_{k+1} \longrightarrow \mathcal{C}_\Gamma$$

Now we show that $\pi_W$ is surjective as follows: Since $\pi_W(I_{k+1}) = 0$ , we have $\operatorname{Ker} \pi_W \supset I_{k+1}$. It follows that the following diagram

$$\begin{array}{ccc} \mathcal{C}_\Sigma & \longrightarrow & \mathcal{C}_\Gamma \\ & \searrow & \uparrow \\ & & \mathcal{C}_\Sigma/I_{k+1} \end{array}$$

commutes and $\dot{\pi_W}(\dot{h}_i) := \pi_W(h_i) = h_i^{'}, i \in W$ is well defined. This shows that $\pi_W(\mathcal{C}_\Sigma)$ is a closed subalgebra in

$\mathcal{C}_\Gamma$ and isomorphic to $\pi_W^{\cdot}(\mathcal{C}_\Sigma/I_{k+1})$. We have $\pi_W^{\cdot}(\mathcal{B}_\Gamma) = \mathcal{I}_\Gamma$. It is clear that $\operatorname{Ker}\pi_W$ is the ideal generated by $h_i$ for $i$ not in $W$ and therefore $\operatorname{Ker}\pi_W^{\cdot}$ is generated by $\dot{h}_i$ for $i$ not in $W$. This comes at once from the definitions of $\pi_W^{\cdot}(\dot{h}_i)$ and $\pi_W(h_i)$ above and the fact that both are equal. We conclude that $\mathcal{B}_\Gamma \operatorname{Ker}\pi_W^{\cdot} = 0$. This again implies that $\mathcal{B}_\Gamma \cap \operatorname{Ker}\pi_W^{\cdot} = 0$. Moreover the following diagram is commutative:

$$\begin{array}{ccc} \mathcal{C}_\Sigma & \longrightarrow & \mathcal{C}_\Gamma \\ \bigcup & & \bigcup \\ \mathcal{B}_\Gamma & \longrightarrow & \mathcal{I}_\Gamma \\ & \searrow & \uparrow \\ & & \mathcal{B}_\Gamma/\operatorname{Ker}\pi_W^{\cdot}. \end{array}$$

So, $\pi_W^{\cdot}(\mathcal{B}_\Gamma)$ is dense and closed in $\mathcal{I}_\Gamma$. Therefore $\pi_W^{\cdot} : \mathcal{B}_\Gamma \longrightarrow \mathcal{I}_\Gamma$ is injective and surjective. □

As a consequence of the above lemma we have the following.

**Proposition 3.** *Let $\mathcal{C}_\triangle$ and $I_k$ defined as above. Then we have an isomorphism*

$$I_k/I_{k+1} \cong \bigoplus_\triangle \mathcal{I}_\triangle,$$

*where the sum is taken over all k-simplexes $\triangle$ in $\Sigma$.*

*Proof.* As in the proof of lemma 1 above with $\Sigma = \triangle$, we find that:

$$I_k/I_{k+1} = \bigoplus_\triangle \mathcal{I}_\triangle.$$

□

In the following we study the $C^*$-algebras $\mathcal{C}_{\Gamma^n}$ associated to simplicial flag complexes $\Gamma$ of a specific simple type. These simplicial complexes is a subcomplex of the "non-commutative spheres" in the sense of Cuntz work (Cuntz 2002). We determine the $K$-theory of $\mathcal{C}_{\Gamma^n}$ and also the $K$-theory of its skeleton filtration. The $K$-theory of $C^*$-algebras is a powerful tool for classifying $C^*$-algebras up to their Projections and unitaries , more details about $K$-theory of $C^*$-algebras found in the references (Blackadar 1986; Murphy 1990; Rørdam et al. 2000; Wegge-Olsen 1993).

We denote by $\Gamma^n$ the simplicial complex with $n+2$ vertices, given in the form

$$V_{\Gamma^n} = \{0^+, 0^-, 1, \ldots, n\},$$

and

$$\Gamma^n = \{\gamma \subset V_{\Gamma^n} \mid \{0^+, 0^-\} \nsubseteq \gamma\}.$$

Let

$$\begin{aligned} \mathcal{C}_{\Gamma^n} = C^*(h_{0^-}, h_{0^+}, h_1, h_2, \ldots, h_n \mid h_{0^-}h_{0^+} \\ = 0, h_i \geqslant 0, \sum_i h_i = 1, \forall i) \end{aligned}$$

be the universal $C^*$- algebra associated to $\Gamma^n$. The existence of such algebras is due to Cuntz (2002). It is clear that for any element $h_i \in \mathcal{C}_{\Gamma^n}$, we have $\|h_i\| \leq 1$.

Denote by $\mathcal{I}$ the natural ideal in $\mathcal{C}_{\Gamma^n}$ generated by products of generators containing all $h_i$, $i \in V_{\Gamma^n}$. Then we have the skeleton filtration

$$\mathcal{C}_{\Gamma^n} = I_0 \supset I_1 \supset I_2 \supset \ldots.. \supset I_{n+1} := \mathcal{I}$$

The aim of this section is to prove that the $K$-theory of the ideals $\mathcal{I}$ in the algebras $\mathcal{C}_{\Gamma^n}$ is equal to zero. We have the following

**Lemma 2.** *Let $\mathcal{C}_{\Gamma^n}$ be as above. Then $\mathcal{C}_{\Gamma^n}$ is homotopy equivalent to $\mathbb{C}$.*

*Proof.* Let $\beta : \mathbb{C} \longrightarrow \mathcal{C}_{\Gamma^n}$ be the natural homomorphism which sends 1 to $1_{\mathcal{C}_{\Gamma^n}}$. For a fixed $i \in V_{\Gamma^n}$ such that $i \neq 0^-, 0^+$, define the homomorphism

$$\alpha : \mathcal{C}_{\Gamma^n} \longrightarrow \mathbb{C}$$

by $\alpha(h_i) = 1$ and $\alpha(h_j) = 0$ for any $j \neq i$. Notice that $\alpha \circ \beta = id_{\mathbb{C}}$. Now define $\varphi_t : \mathcal{C}_{\Gamma^n} \longrightarrow \mathcal{C}_{\Gamma^n}$, $h_i \longmapsto h_i + (1-t)(\sum_{j\neq i} h_j)$, $h_j \longmapsto t(h_j) j \in V_{\Gamma^n} \setminus \{i\}$. The elements $\varphi_t(h_j)$, $j \in V_{\Gamma^n}$, satisfy the same relations as the elements $h_j$ in $\mathcal{C}_{\Gamma^n}$:

(i) $\varphi_t(h_j) \geq 0$

(ii) $$\begin{aligned} \varphi_t\left(\sum_j h_j\right) &= \varphi_t(h_i) + \sum_{j\neq i} \varphi_t(h_j) \\ &= h_i + (1-t)\left(\sum_{j\neq i} h_j\right) + t\left(\sum_{j\neq i} h_j\right) \\ &= h_i + \sum_{j\neq i} h_j \text{ for fixed } i \\ &= \sum_j h_j = 1 \text{ for all } j, \end{aligned}$$

(iii) $\varphi_t(h_{0^-})\varphi_t(h_{0^+}) = t^2(h_{0^-}h_{0^+}) = 0.$

We note that $\varphi_1 = id_{\mathcal{C}_{\Gamma^n}}$ and $\varphi_0 = \beta \circ \alpha$.

This implies that

$$\varphi_0 = \beta \circ \alpha \sim Id_{\mathcal{C}_{\Gamma^n}}.$$

This means that $\mathcal{C}_{\Gamma^n}$ is homotopy equivalent to $\mathbb{C}$. □

From the above lemma , we have $K_*(\mathcal{C}_{\Gamma^n}) = K_*(\mathbb{C})$, for $* = 0, 1$.

Now we describe the subquotients of the skeleton filtration in $\mathcal{C}_{\Gamma^n}$.

**Proposition 4.** *In the $C^*$-algebra $\mathcal{C}_{\Gamma^n}$ one has*

$$I_k/I_{k+1} \cong \bigoplus_{\triangle} \mathcal{I}_{\triangle} \oplus \bigoplus_{\gamma} \mathcal{I}_{\gamma} \ ,$$

*where the sum is taken over all subcomplexes $\triangle$ of $\Gamma^n$ which are isomorphic to the standard $k$-simplex $\triangle$ and over all subcomplexes $\gamma$ of $\Gamma^n$ which contain both vertices $0^+, 0^-$ and the second sum is taken over every subcomplex $\gamma$ which contains both vertices $0^+, 0^-$ and whose number of vertices is $k+1$.*

*Proof.* We use Lemma 1 above. For every $W \subset V_{\Gamma^n}$ with $|W| = k+1$, we have two cases. Either $\{0^+, 0^-\}$ is not a subset of $W$, then $\Gamma$ is a $k$- simplex, or $\{0^+, 0^-\}$ is a subset of $W$, then $\Gamma$ is a subcomplex in $\Gamma^n$ isomorphic to $\gamma$. This proves our proposition. □

**Lemma 3.** *For the complex $\Gamma^n$ with $n+2$ vertices, $\mathcal{C}_{\Gamma^n}/I_1$ is commutative and isomorphic to $\mathbb{C}^{n+2}$.*

*Proof.* Let $\dot{h}_i$ denote the image of a generator $h_i$ for $\mathcal{C}_{\Gamma^n}$. One has the following relations:

$$\sum_i \dot{h}_i = 1, \quad \dot{h}_i \dot{h}_j = 0, \quad i \neq j.$$

For every $\dot{h}_i$ in $\mathcal{C}_{\Gamma^n}/I_1$ we have

$$\dot{h}_i = \dot{h}_i \left( \sum_i \dot{h}_i \right) = \dot{h}_i^2.$$

Hence $\mathcal{C}_{\Gamma^n}/I_1$ is generated by $n+2$ different orthogonal projections and therefore $\mathcal{C}_{\Gamma^n}/I_1 \cong \mathbb{C}^{n+2}$. □

**Lemma 4.** *$I_1/I_2$ in $\mathcal{C}_{\Gamma^n}$ is isomorphic to $I_1^{ab}/I_2^{ab}$ in $\mathcal{C}^{ab}_{\Gamma^n}$.*

*Proof.* From the proposition 4 above, one has

$$I_1/I_2 \cong \bigoplus_{\triangle^1} \mathcal{I}_{\triangle^1}$$

where $\triangle^1$ is 1-simplex, and

$$I_1^{ab}/I_2^{ab} \cong \bigoplus_{\triangle^1} \mathcal{I}^{ab}_{\triangle^1}.$$

Since $\mathcal{I}_{\triangle^1} \subset \mathcal{C}_{\triangle^1}$ is commutative because the generators of $\mathcal{C}_{\triangle^1}$ commute (since $h_{s_1} = 1 - h_{s_0}$). We get

$$\mathcal{I}_{\triangle^1} \cong \mathcal{I}^{ab}_{\triangle^1} \cong C_0(0,1).$$

□

**Lemma 5.** *In $\mathcal{C}_{\Gamma^n}$ , we have $K_0(I_1/I_2) = 0$ and $K_1(I_1/I_2) = \mathbb{Z}^{\binom{n}{2}+2n}$.*

*Proof.* By applying above lemma , and proposition 4, we have

$$I_1/I_2 \cong \bigoplus_{\triangle^1} \mathcal{I}_{\triangle^1}$$

The sum contain $\binom{n}{2} + 2n$ 1-simplex, $\triangle^1 \cong C_0(0,1)$. where $K_0(C_0(0,1)) = 0$ and $K_1(C_0(0,1)) = \mathbb{Z}$. □

**Lemma 6.** *$\mathcal{C}_{\Gamma^n}/I_2$ is a commutative $C^*$-algebra.*

*Proof.* Consider the extension

$$0 \longrightarrow I_1/I_2 \longrightarrow \mathcal{C}_{\Gamma^n}/I_2 \longrightarrow \mathcal{C}_{\Gamma^n}/I_1 \longrightarrow 0$$

and the analogous extension for the abelianized algebras.

The extensions above induce the following commutative diagram:

$$\begin{array}{ccccccccc} 0 \longrightarrow & I_1/I_2 & \longrightarrow & \mathcal{C}_{\Gamma^n}/I_2 & \longrightarrow & \mathcal{C}_{\Gamma^n}/I_1 & \longrightarrow 0 \\ & \downarrow & & \downarrow & & \downarrow & \\ 0 \longrightarrow & I_1^{ab}/I_2^{ab} & \longrightarrow & \mathcal{C}^{ab}_{\Gamma^n}/I_2^{ab} & \longrightarrow & \mathcal{C}^{ab}_{\Gamma^n}/I_1^{ab} & \longrightarrow 0 \end{array}$$

We have from 3 isomorphisms $\mathcal{C}_{\Gamma^n}/I_1 \cong \mathcal{C}^{ab}_{\Gamma^n}/I_1^{ab} \cong \mathbb{C}^{n+2}$ and from 4 that $I_1/I_2 \cong I_1^{ab}/I_2^{ab}$, so

$$\mathcal{C}_{\Gamma^n}/I_2 \cong \mathcal{C}^{ab}_{\Gamma^n}/I_2^{ab}.$$

□

**Lemma 7.** *$C^*$-algebra $\mathcal{C}_{\Gamma^1}$ is commutative and $K_*(I_2) = 0$, $* = 0, 1$ where $I_2$ is an ideal in $\mathcal{C}_{\Gamma^1}$ defined as in the above.*

*Proof.* $\mathcal{C}_{\Gamma^1}$ is generated by three positive generators, $h_{0^-}, h_{0^+}, h_1$. Consider the product of two generators, say $h_1 h_{0^-}$. We have that $1, h_{0^-}$ and $h_{0^+}$ commute with $h_{0^-}$, therefore also $h_1 = 1 - h_{0^-} - h_{0^+}$.

By a similar computation we can show that $h_{0^+}$ and $h_1$ commute. This implies that $\mathcal{C}_{\Gamma^1}$ is commutative. Therefore $I_2 = 0$ in $\mathcal{C}_{\Sigma^1}$ Then, at once $K_*(I_2) = 0$. □

**Competing interests**

The author declare that he has no competing interests.

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**References**

Blackadar B (1986) K-theory for operator algebras. MSRI Publ. 5, Cambridge University Press

Cuntz J (1981) K-theory for certain $C^*$-algebras. Ann Math 113:181–197

Cuntz J (1984) K-theory and $C^*$-algebras. Proc. Conf. on K-theory (Bielefeld,1982). Springer Lecture Notes in Math. 1046:55–79

Cuntz J (1993) A survey of some aspects of noncommutative geometry. Jahresber D Dt Math-Verein 95:60–84

Cuntz J (2002) Non-commutative simplicial complexes and Baum-Connes-conjecture. GAFA, Geom Func Anal 12: 307–329

Davidson KR (1996) $C^*$-algebras by example. Fields Institute monographs, Amer. Math. Soc, Providence

Murphy GJ (1990) $C^*$-algebras and operator theory. Academic Press

Omran S (2005) $C^*$-algebras associated with higher-dimensional noncommutative simplicial complexes and their $K$-theory. Dissertation, Münster: Univ. Münster-Germany

Omran S (2013) $C^*$-algebras associated noncommutative circle and their $K$-theory. Aust J Math Anal Appl 10(1): 1–8. Article 8
Omran S, Gouda GhY (2012) On the K-theory of $C^*$-algebras associated with $n$-Simplexes. Int J Math Anal 6(17): 847–855
Rørdam M, Larsen F, Laustsen N (2000) An introduction to K-theory for $C^*$-algebras. London Mathematical Society Student Text 49, Cambridge University Press
Wegge-Olsen NE (1993) K-theory and $C^*$-algebras. Oxford University Press, New York

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